

Algorithms and Computational Thinking

Autumn 2017

Tuesday, 21 December 2017

Probabilistic Algorithms

Markov Chain

Markov chains are mathematical systems that jump from one "state" (a situation or set of values) to another. For example, if you made a Markov chain model of your movement in the city of Lausanne, you might consider moving through different stations like "Lausanne-Flon", "Lausanne-Gare", "Bel-Air" etc. Here, each state in the Markov chain can be considered as your position at one of the above stations and thus the state changes (transition) indicate your movement from one station to another. A sample representation of the above can be shown as below.



As shown in the figure, the Markov Chain model is an oriented graph in which the sum of all outgoing probabilities (percentages) of each state is equal to 1.0 (100%). In the above representation some of the transitions are

omitted as they are insignificant. A Markov Chain can be used to predict the most likely next state from a current state. Considering the above model is the representation of a person named "Bob", his most likely next station from Bel-Air is Flon because the transition probability for above is .57 (57%).

In this exercise, you will consider only three stations (i.e., three states) where Bob moves : Flon is represented by number "0", Gare by number "1" and Bel-Air by number "2".

1. In the first part of the exercise, you will create Bob's Markov Chain model in terms of state-transition matrix by instantiating the model with the below state sequence (i.e. history of Bob's past movements across the three stations).

(1,2,0,1,1,1,2,0,1,2,1,2,0,1,2,1,0,0,0,1,2,1,2,1,0,2,0,2,1,2,1,2,1,1,1,0,1,0,1,0,1,0)

Here, (1,2,0...) means that Bob moves from Gare to Bel-Air and then to Flon.

2. In the second part of this exercise, you have to estimate Bob's most likely next station, considering his current station is Flon.

Note : You can choose between Python or Scala for the above implementation.