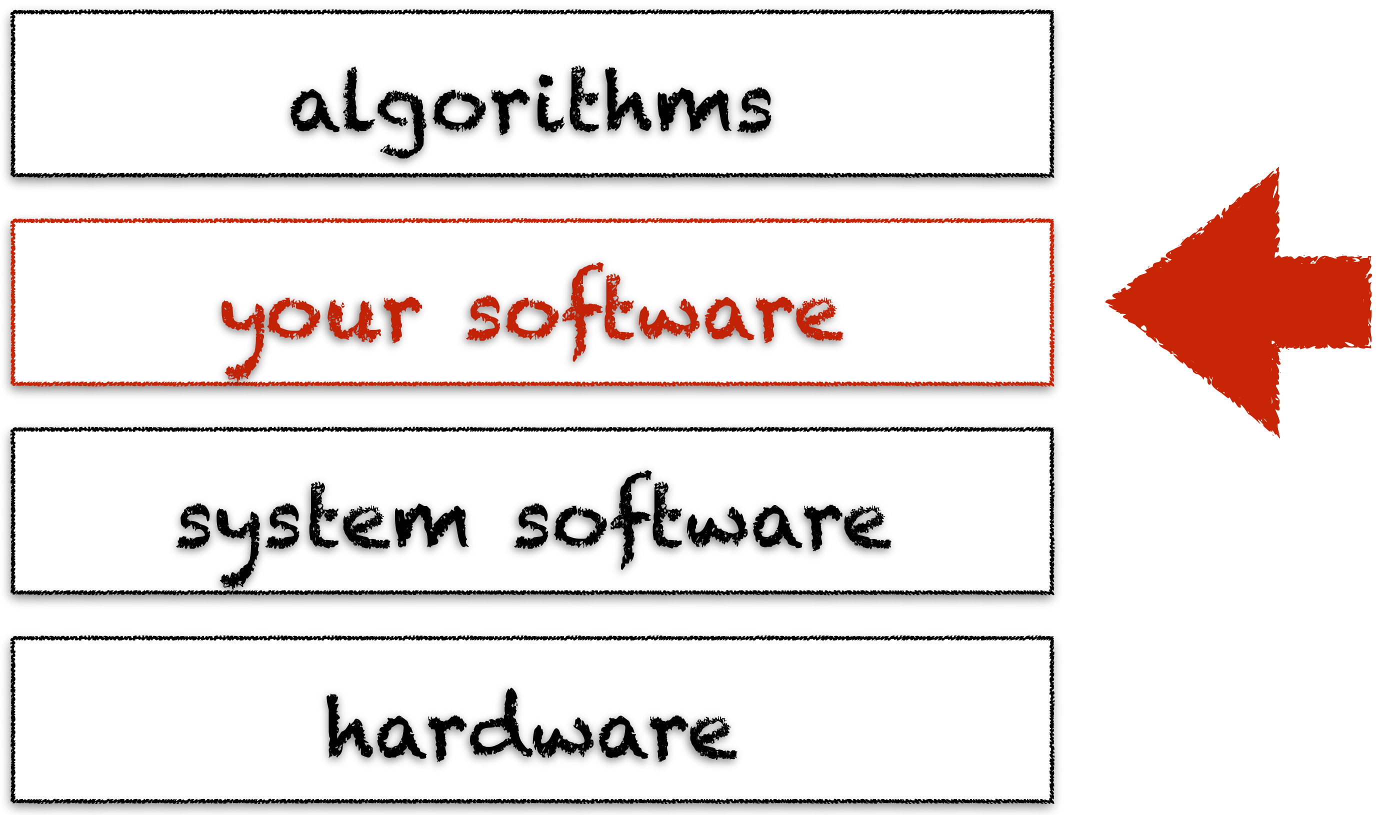
abstract classes & types

learning objectives



- learn how to define and use abstract classes
- learn how to define types without implementation
- learn about multiple inheritance of types

a mathematical example

representing matrices

a **rectangular array** of elements arranged **in rows and columns**

a_{ij}

n columns

j changes

m rows

i changes

$$\begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{bmatrix}$$

$m \times n$ *matrix*

i designates the row
j designates the column

examples

$$\begin{bmatrix} 4 \\ 1 \\ 8 \end{bmatrix} \qquad \begin{bmatrix} 3 & 7 & 2 \end{bmatrix} \qquad \begin{bmatrix} 9 & 13 & 5 \\ 1 & 11 & 7 \\ 2 & 6 & 3 \end{bmatrix}$$

3×1 *matrix*   1×3 *matrix*   3×3 *matrix*

---

matrices are used in many branches of physics, math, computer graphics, etc.

linear equation

$$a_{11}x_1 + a_{12}x_2 = b_1$$
$$a_{21}x_1 + a_{22}x_2 = b_2$$

$$A\vec{x} = \vec{b} \quad \text{where} \quad A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \quad \vec{x} = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \quad \vec{b} = \begin{bmatrix} b_1 \\ b_2 \end{bmatrix}$$

representing matrices

sparse matrix

one with most of its elements equal to zero

dense matrix

one with most of its elements not equal to zero

square matrix

one with equal number of rows and columns

diagonal matrix

one with all off-diagonal elements equal to zero

$$d_{i,j} = 0 \text{ if } i \neq j \; \forall i, j \in \{1, 2, \ldots, n\}$$

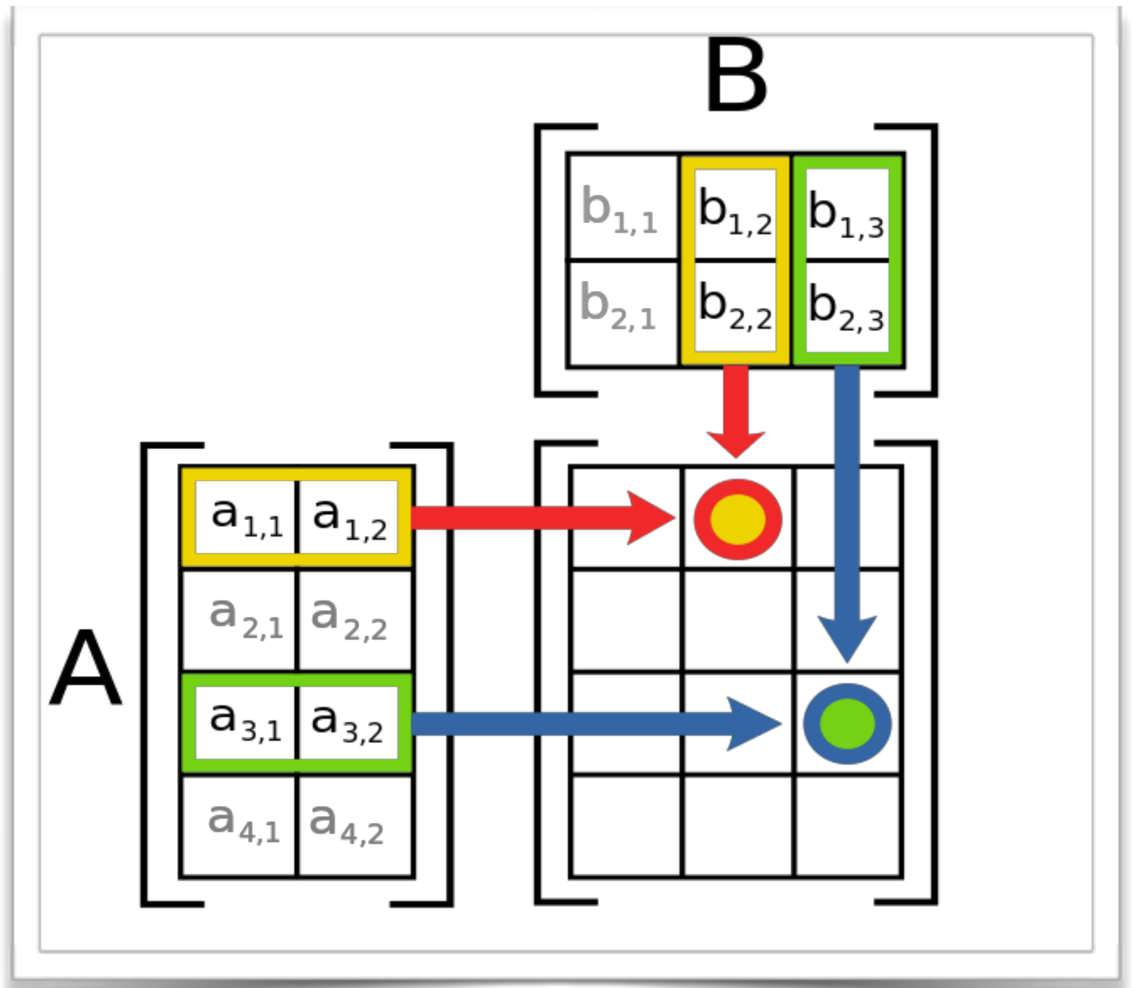
addition

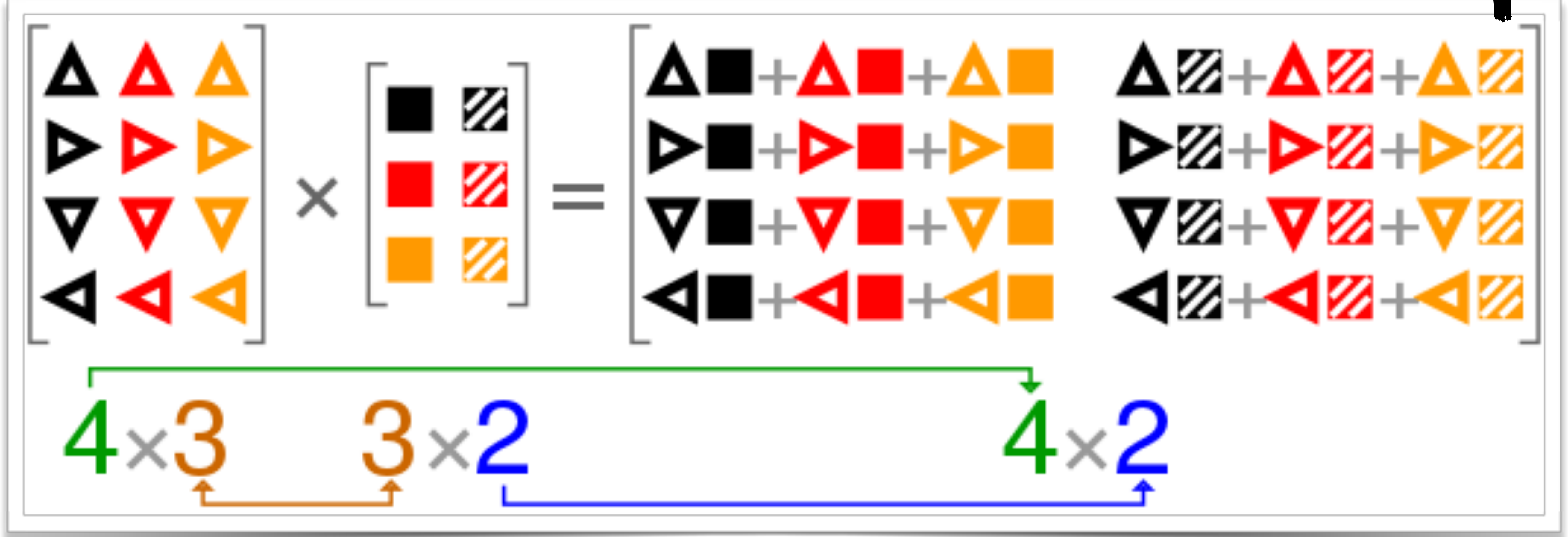
$$\mathbf{A} + \mathbf{B} = \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{bmatrix} + \begin{bmatrix} b_{11} & b_{12} & \cdots & b_{1n} \\ b_{21} & b_{22} & \cdots & b_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ b_{m1} & b_{m2} & \cdots & b_{mn} \end{bmatrix} = \begin{bmatrix} a_{11}+b_{11} & a_{12}+b_{12} & \cdots & a_{1n}+b_{1n} \\ a_{21}+b_{21} & a_{22}+b_{22} & \cdots & a_{2n}+b_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{m1}+b_{m1} & a_{m2}+b_{m2} & \cdots & a_{mn}+b_{mn} \end{bmatrix}$$

$$\begin{bmatrix} 1 & 3 \\ 1 & 0 \\ 1 & 2 \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 7 & 5 \\ 2 & 1 \end{bmatrix} = \begin{bmatrix} 1+0 & 3+0 \\ 1+7 & 0+5 \\ 1+2 & 2+1 \end{bmatrix}$$

A and B must have the same number of rows and columns

representing matrices
multiplication





$$\mathbf{A} = \begin{pmatrix} A_{11} & A_{12} & \cdots & A_{1m} \\ A_{21} & A_{22} & \cdots & A_{2m} \\ \vdots & \vdots & \ddots & \vdots \\ A_{n1} & A_{n2} & \cdots & A_{nm} \end{pmatrix} \quad \mathbf{B} = \begin{pmatrix} B_{11} & B_{12} & \cdots & B_{1p} \\ B_{21} & B_{22} & \cdots & B_{2p} \\ \vdots & \vdots & \ddots & \vdots \\ B_{m1} & B_{m2} & \cdots & B_{mp} \end{pmatrix} \quad \mathbf{AB} = \begin{pmatrix} (\mathbf{AB})_{11} & (\mathbf{AB})_{12} & \cdots & (\mathbf{AB})_{1p} \\ (\mathbf{AB})_{21} & (\mathbf{AB})_{22} & \cdots & (\mathbf{AB})_{2p} \\ \vdots & \vdots & \ddots & \vdots \\ (\mathbf{AB})_{n1} & (\mathbf{AB})_{n2} & \cdots & (\mathbf{AB})_{np} \end{pmatrix}$$

$$(\mathbf{AB})_{ij} = \sum_{k=1}^{m} A_{ik} B_{kj}$$

$$\mathbf{A} = \begin{pmatrix} a & b & c \\ p & q & r \\ u & v & w \end{pmatrix} \quad \mathbf{B} = \begin{pmatrix} \alpha & \beta & \gamma \\ \lambda & \mu & \nu \\ \rho & \sigma & \tau \end{pmatrix}$$

$$\mathbf{AB} = \begin{pmatrix} a & b & c \\ p & q & r \\ u & v & w \end{pmatrix} \begin{pmatrix} \alpha & \beta & \gamma \\ \lambda & \mu & \nu \\ \rho & \sigma & \tau \end{pmatrix} = \begin{pmatrix} a\alpha + b\lambda + c\rho & a\beta + b\mu + c\sigma & a\gamma + b\nu + c\tau \\ p\alpha + q\lambda + r\rho & p\beta + q\mu + r\sigma & p\gamma + q\nu + r\tau \\ u\alpha + v\lambda + w\rho & u\beta + v\mu + w\sigma & u\gamma + v\nu + w\tau \end{pmatrix}$$

$$\mathbf{BA} = \begin{pmatrix} \alpha & \beta & \gamma \\ \lambda & \mu & \nu \\ \rho & \sigma & \tau \end{pmatrix} \begin{pmatrix} a & b & c \\ p & q & r \\ u & v & w \end{pmatrix} = \begin{pmatrix} \alpha a + \beta p + \gamma u & \alpha b + \beta q + \gamma v & \alpha c + \beta r + \gamma w \\ \lambda a + \mu p + \nu u & \lambda b + \mu q + \nu v & \lambda c + \mu r + \nu w \\ \rho a + \sigma p + \tau u & \rho b + \sigma q + \tau v & \rho c + \sigma r + \tau w \end{pmatrix}$$

$$\mathbf{AB} \neq \mathbf{BA}$$

representing matrices

```
class Matrix(private val rows: Int, private val cols: Int) {
 if (rows <= 0)
   throw new IllegalArgumentException("A matrix must have a positive number of rows")
  if (cols <= 0)
   throw new IllegalArgumentException("A matrix must have a positive number of columns")

  private var matrix = Array.ofDim[Int](rows, cols)

  def this(rows: Int, cols: Int, mtx: Array[Array[Int]]) {
    this(rows,cols)
    for (i <- 0 to rows - 1)
      mtx(i).copyToArray(matrix(i))
  }

  def numOfRows: Int = this.rows
  def numOfCols: Int = this.cols

  def apply(i: Int, j: Int): Int = matrix(i)(j)
  def update(i: Int, j: Int, v: Int) = matrix(i)(j) = v


  def print: Unit = {
    println(this.getClass.getSimpleName)
    for (i <- 0 to this.numOfRows - 1) {
      Console.print("|  ")
      for (j <- 0 to this.numOfCols - 1) {
        Console.print(s"${this(i,j)}  ")
      }
      println("|")
    }
  }
...
```

```
var m = new Matrix(3,3)
m.print()

val mtx : Array[Array[Int]] = Array( Array(2, 4, 6) , Array(3, 6, 9))

m = new Matrix(2,3,mtx)
m.print()
```

```
Matrix
|  0  0  0  |
|  0  0  0  |
|  0  0  0  |
Matrix
|  2  4  6  |
|  3  6  9  |
```

in the following, we assume that indices go from 0 to $n{-}1$ rather than 1 to n

representing matrices

```scala
class Matrix(private val rows: Int, private val cols: Int) {
 if (rows <= 0)
    throw new IllegalArgumentException("A matrix must have a positive number of rows")
  if (cols <= 0)
    throw new IllegalArgumentException("A matrix must have a positive number of columns")

  private var matrix = Array.ofDim[Int](rows, cols)

  def this(rows: Int, cols: Int, mtx: Array[Array[Int]]) {
    this(rows,cols)
    for (i <- 0 to rows - 1)
      mtx(i).copyToArray(matrix(i))
  }

  def numOfRows: Int = this.rows
  def numOfCols: Int = this.cols

  def apply(i: Int, j: Int): Int = matrix(i)(j)
  def update(i: Int, j: Int, v: Int) = matrix(i)(j) = v

  def print: Unit = {
    println(this.getClass.getSimpleName)
    for (i <- 0 to this.numOfRows - 1) {
      Console.print("|  ")
      for (j <- 0 to this.numOfCols - 1) {
        Console.print(s"${this(i,j)}  ")
      }
      println("|")
    }
  }
...
```

```scala
val mtx : Array[Array[Int]] = Array( Array(2, 4, 6) , Array(3, 6, 9))
m = new Matrix(2,3,mtx)
m.print()

println(s"m(0,0) = ${m(0,0)}"); m(0,0) = 7; println(s"m(0,0) = ${m(0,0)}")
m.print()
```

```
Matrix
|  2  4  6  |
|  3  6  9  |
m(0,0) = 2
m(0,0) = 7
Matrix
|  7  4  6  |
|  3  6  9  |
```

representing matrices

```scala
class Matrix(private val rows: Int, private val cols: Int) {
  ...

  def +(other: Matrix): Matrix = {
    if (this.numOfRows != other.numOfRows || this.numOfCols != other.numOfCols)
      throw new IllegalArgumentException("Matrices must have the same number of rows" + " and columns when added")

    val result = new Matrix(this.numOfRows, this.numOfCols)

    for (i <- 0 to this.numOfRows - 1; j <- 0 to this.numOfCols - 1)
      result(i,j) = this (i, j) + other(i, j)
    return result
  }

  def *(other: Matrix): Matrix = {
    if (this.numOfCols != other.numOfRows)
      throw new IllegalArgumentException("The number of columns in the first matrix must be" +
                          "equal to the number of rows of the second matrix when multiplied")

    val result = new Matrix(this.numOfRows, other.numOfCols)

    for (i <- 0 to result.numOfRows - 1; j <- 0 to result.numOfCols - 1) {
      var entry: Int = 0
      for (k <- 0 to this.numOfCols - 1)
        entry = entry + this (i, k) * other(k, j)
      result(i,j) = entry
    }
    return result
  }
}
```

$$(\mathbf{AB})_{ij} = \sum_{k=1}^{m} A_{ik} B_{kj}$$

sparse matrices

```
class SparseMatrix(private val rows: Int, private val cols: Int) {
 if (rows <= 0)
   throw new IllegalArgumentException("A matrix must have a positive number of rows")
  if (cols <= 0)
   throw new IllegalArgumentException("A matrix must have a positive number of columns")

  private var map = scala.collection.mutable.Map[(Int,Int),Int]()

  def this(rows: Int, cols: Int, mtx: Map[(Int,Int),Int]) {
    this(rows,cols)
    map = collection.mutable.Map(mtx.toSeq: _*)
  }

  def numOfRows: Int = this.rows
  def numOfCols: Int = this.cols

  def apply(i: Int, j: Int): Int = map.getOrElse((i,j),0)
  def update(i: Int, j: Int, v: Int): Unit = this.map((i,j)) = v

  def print: Unit = {
    println(this.getClass.getSimpleName)
    for (i <- 0 to this.numOfRows - 1) {
      Console.print("|  ")
      for (j <- 0 to this.numOfCols - 1) {
        Console.print(s"${this(i,j)}  ")
      }
      println("|")
    }
  }
 ...
}
```

```
var m = new Matrix(3,3)
m.print()
```

```
SparseMatrix
|  0  0  0  |
|  0  0  0  |
|  0  0  0  |
```

```
val mtx = scala.collection.immutable.Map((6,6) -> 6, (3,3) -> 3)
m = new Matrix(10,10, mtx)
m.print()
```

```
SparseMatrix
|  0  0  0  0  0  0  0  0  0  0  |
|  0  0  0  0  0  0  0  0  0  0  |
|  0  0  0  0  0  0  0  0  0  0  |
|  0  0  0  3  0  0  0  0  0  0  |
|  0  0  0  0  0  0  0  0  0  0  |
|  0  0  0  0  0  0  0  0  0  0  |
|  0  0  0  0  0  0  6  0  0  0  |
|  0  0  0  0  0  0  0  0  0  0  |
|  0  0  0  0  0  0  0  0  0  0  |
|  0  0  0  0  0  0  0  0  0  0  |
```

problem

```scala
class Matrix(private val rows: Int, private val cols: Int) {
  if (rows <= 0)
    throw new IllegalArgumentException("A matrix must have a positive num
  if (cols <= 0)
    throw new IllegalArgumentException("A matrix must have a positive num

  private var matrix = Array.ofDim[Int](rows, cols)

  def this(rows: Int, cols: Int, mtx: Array[Array[Int]]) {
    this(rows,cols)
    for (i <- 0 to rows - 1)
      mtx(i).copyToArray(matrix(i))
  }

  def numOfRows: Int = this.rows
  def numOfCols: Int = this.cols

  def apply(i: Int, j: Int): Int = matrix(i)(j)
  def update(i: Int, j: Int, v: Int) = matrix(i)(j) = v

  def print: Unit = {
    println(this.getClass.getSimpleName)
    for (i <- 0 to this.numOfRows - 1) {
      Console.print("|  ")
      for (j <- 0 to this.numOfCols - 1) {
        Console.print(s"${this(i,j)}  ")
      }
      println("|")
    }
  }
  ...
```

```scala
class SparseMatrix(private val rows: Int, private val cols: Int) {
  if (rows <= 0)
    throw new IllegalArgumentException("A matrix must have a positive number of rows")
  if (cols <= 0)
    throw new IllegalArgumentException("A matrix must have a positive number of columns")

  private var map = scala.collection.mutable.Map[(Int,Int),Int]()

  def this(rows: Int, cols: Int, mtx: Map[(Int,Int),Int]) {
    this(rows,cols)
    map = collection.mutable.Map(mtx.toSeq: _*)
  }

  def numOfRows: Int = this.rows
  def numOfCols: Int = this.cols

  def apply(i: Int, j: Int): Int = map.getOrElse((i,j),0)
  def update(i: Int, j: Int, v: Int): Unit = this.map((i,j)) = v

  def print: Unit = {
    println(this.getClass.getSimpleName)
    for (i <- 0 to this.numOfRows - 1) {
      Console.print("|  ")
      for (j <- 0 to this.numOfCols - 1) {
        Console.print(s"${this(i,j)}  ")
      }
      println("|")
    }
  }
  ...
```

code duplication … again!

problem

can we blend dense and sparse matrices?

```
def *(other: Matrix): Matrix = {
   if (this.numOfCols != other.numOfRows)
     throw new IllegalArgumentException("The number of columns in the first matrix must be" +
                         "equal to the number of rows of the second matrix when multiplied")

   val result = new Matrix(this.numOfRows, other.numOfCols)

   for (i <- 0 to result.numOfRows - 1; j <- 0 to result.numOfCols - 1) {
     var entry: Int = 0
     for (k <- 0 to this.numOfCols - 1)
       entry = entry + this (i, k) * other(k, j)
     result(i,j) = entry
   }
   return result
 }
```

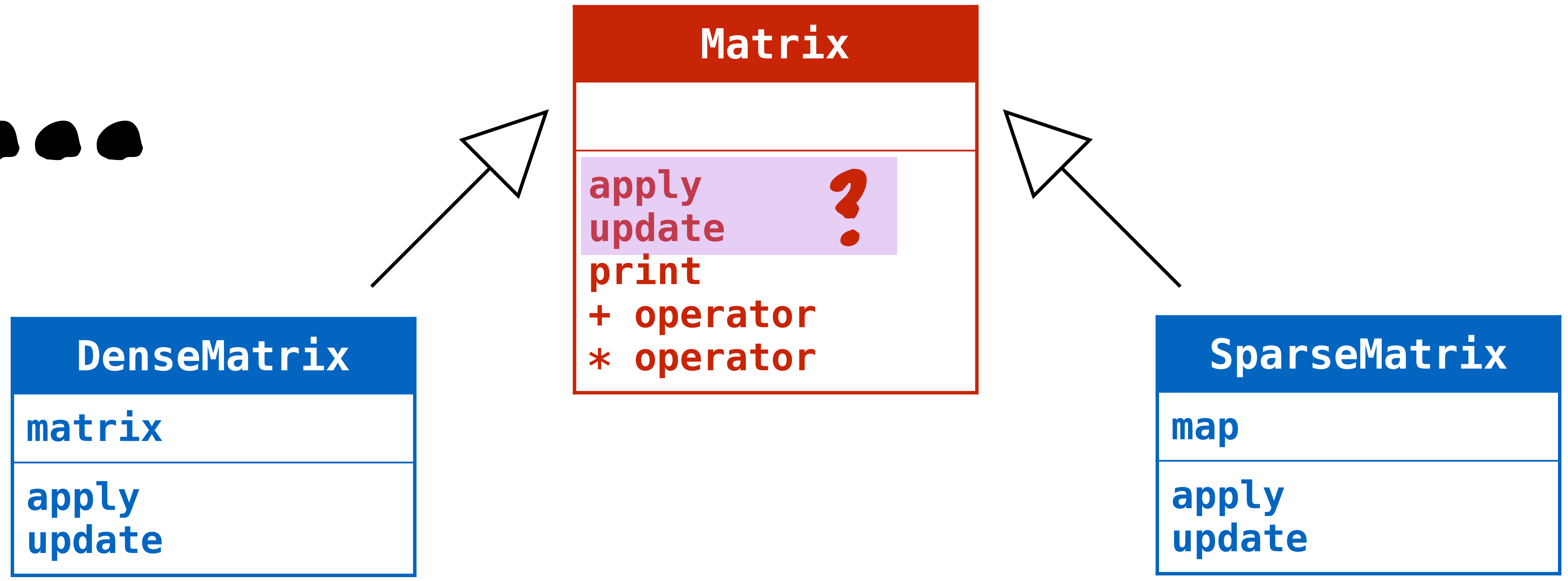
```
def *(other: SparseMatrix): SparseMatrix = {
   if (this.numOfCols != other.numOfRows)
     throw new IllegalArgumentException("The number of columns in the first matrix must be" +
                         "equal to the number of rows of the second matrix when multiplied")

   val result = new SparseMatrix(this.numOfRows, other.numOfCols)

   for (i <- 0 to result.numOfRows - 1; j <- 0 to result.numOfCols - 1) {
     var entry: Int = 0
     for (k <- 0 to this.numOfCols - 1)
       entry = entry + this (i, k) * other(k, j)
     result(i,j) = entry
   }
   return result
 }
```

incompatible types

solution

inheritance

yes, but...



the superclass delegates the internal representation to its subclasses

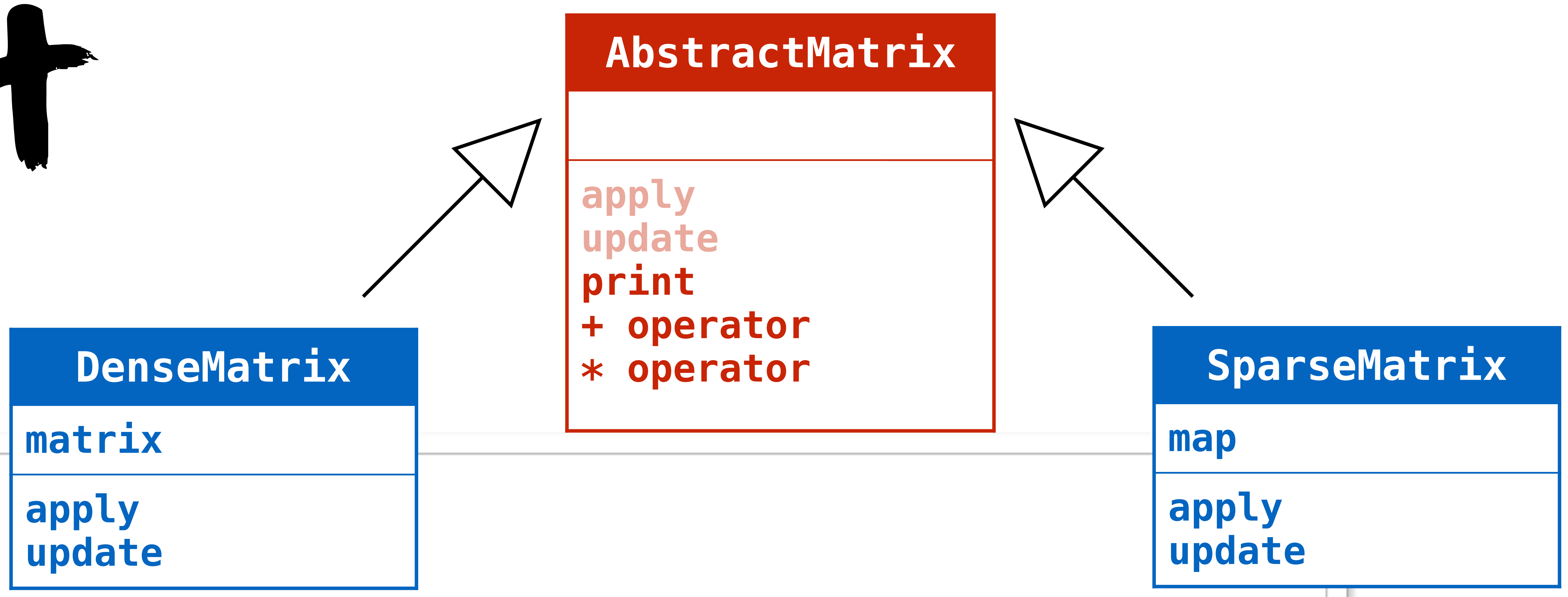
so methods `apply` and `update` have no default or shared implementation

these methods are abstract in the superclass

solution

abstract class

abstract class

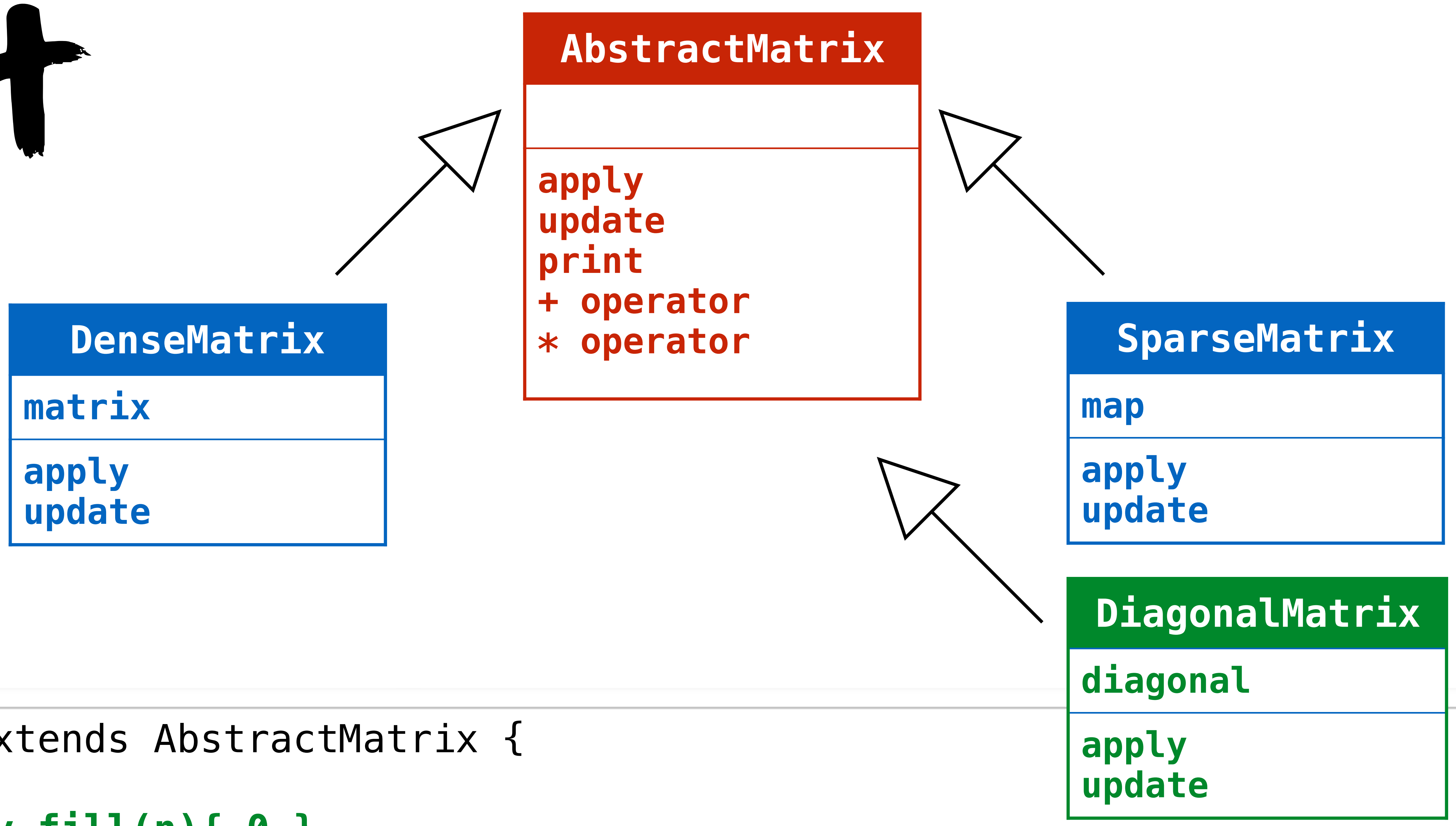



```
abstract class AbstractMatrix {
  def apply(i: Int, j: Int): Int
  def update(i: Int, j: Int, v: Int)

  def +(other: AbstractMatrix): AbstractMatrix = {
    if (this.numOfRows != other.numOfRows || this.numOfCols != other.numOfCols)
      throw new IllegalArgumentException("Matrices must have the same number of rows and columns when added")

    val result = new DenseMatrix(this.numOfRows, this.numOfCols)
    for (i <- 0 to this.numOfRows - 1; j <- 0 to this.numOfCols - 1)
      result(i,j) = this (i, j) + other(i, j)
    return result
  }

  def *(other: AbstractMatrix): AbstractMatrix = {
    if (this.numOfCols != other.numOfRows)
      throw new IllegalArgumentException("The number of columns in the first matrix must be equal" +
                                         " to the number of rows of the second matrix when multiplied")

    val result = new DenseMatrix(this.numOfRows, other.numOfCols)
    for (i <- 0 to result.numOfRows - 1; j <- 0 to result.numOfCols - 1) {
      var entry: Int = 0
      for (k <- 0 to this.numOfCols - 1)
        entry = entry + this (i, k) * other(k, j)
      result(i,j) = entry
    }
    return result
  }
 ...
}
```

why use DenseMatrix, not SparseMatrix?

could we come with a better scheme?

abstract class




```
class DiagonalMatrix(private val n: Int) extends AbstractMatrix {

  private var diagonal : Array[Int] = Array.fill(n){ 0 }

  def this(n: Int, diag: Array[Int]) {
    this(n)
    this.diagonal = diag
  }

  def apply(i: Int, j: Int): Int = if (i != j) 0 else diagonal(i)

  def update(i: Int, j: Int, v: Int): Unit = {
    if (i == j)
      this.diagonal(i) = v
    else (i != j && v != 0)
      throw new IllegalArgumentException("A diagonal matrix should only" + " have zeros outside its diagonal")
  }
}
```

what if all methods could be abstract?

identity matrix

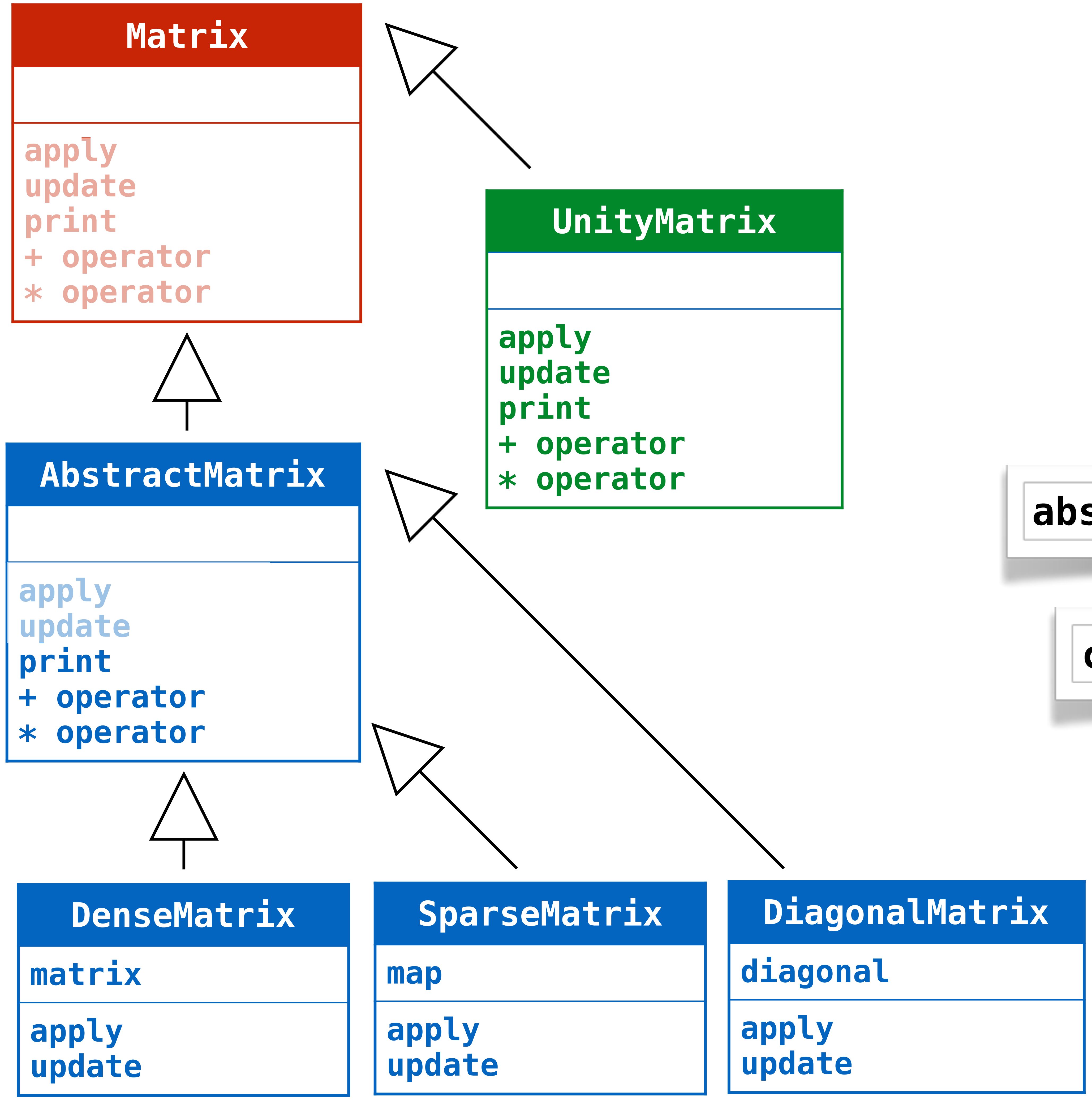
$$A \times I_n = I_n \times A = A$$

$$\begin{bmatrix} 1 & 0 & 0 & \cdots & 0 \\ 0 & 1 & 0 & \cdots & 0 \\ 0 & 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \cdots & 1 \end{bmatrix} = I_n$$

```
class IdentityMatrix(private val n: Int) extends AbstractMatrix {
  def apply(i: Int, j: Int): Int = if (i == j) 1 else 0
  def update(i: Int, j: Int, v: Int): Unit = {
    if (i != j && v != 1)
      throw new IllegalArgumentException("A unity matrix can only have ones on its diagonal")
  }

  override def *(other: Matrix): Matrix = other.duplicate

  override def +(other: Matrix): Matrix = {
    val result = other.duplicate
    for (i <- 0 to n - 1)
      result(i,i) = result(i,i) + 1
    return result
  }
}
```


solution
traits

traits




```scala
trait Matrix {
  def numOfRows(): Int
  def numOfCols(): Int

  def apply(i: Int, j: Int): Int
  def update(i: Int, j: Int, v: Int)

  def +(other: Matrix): Matrix
  def *(other: Matrix): Matrix

  def print
}
```

```scala
abstract class AbstractMatrix extends Matrix { ... }
```

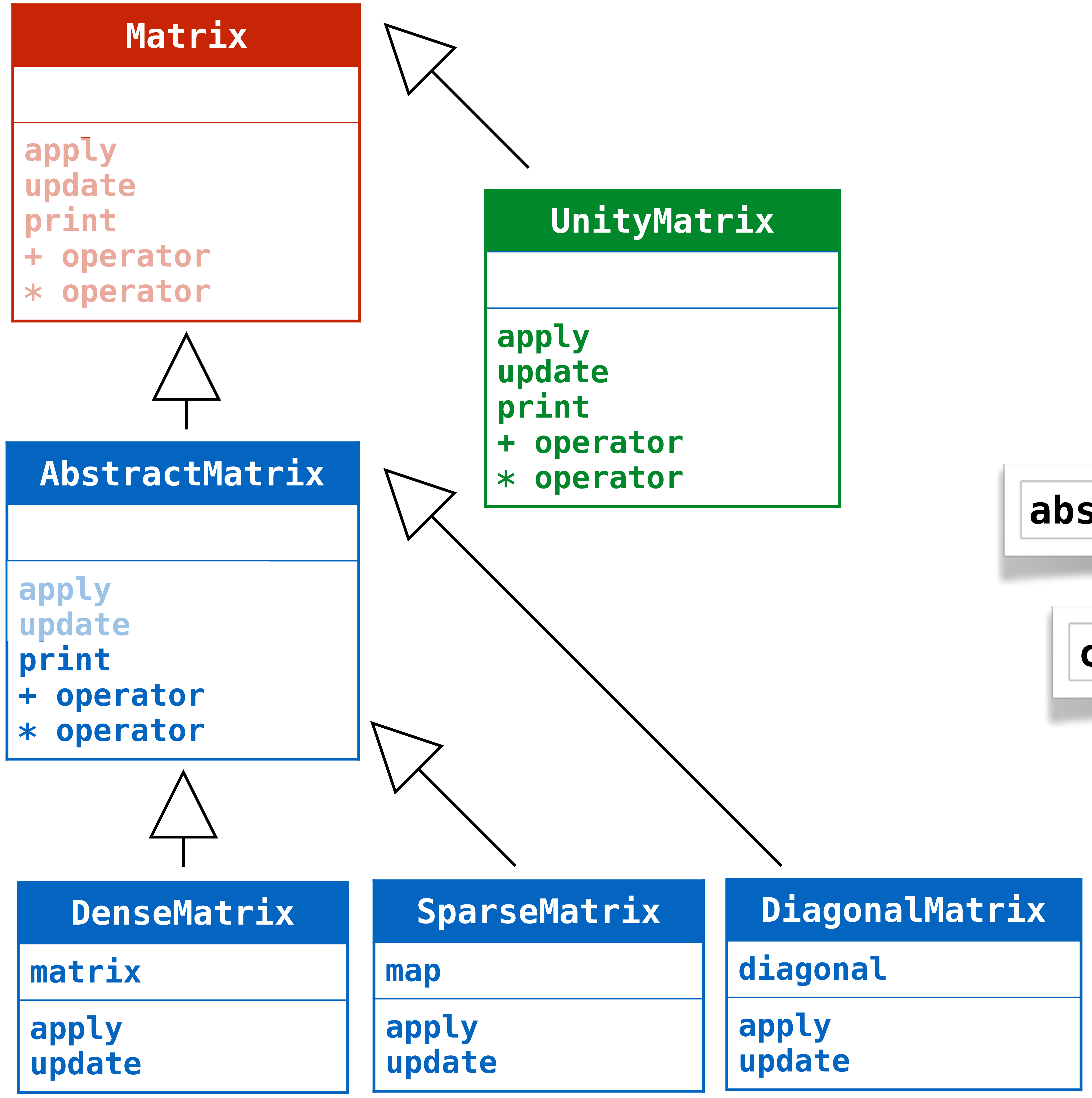
```scala
class DenseMatrix(...) extends AbstractMatrix { ... }
```

```scala
class SparseMatrix(...) extends AbstractMatrix { ... }
```

```scala
class DiagonalMatrix(...) extends AbstractMatrix { ... }
```

```scala
class UnityMatrix(...) extends Matrix { ... }
```

interface




```
public interface Matrix {
    int numOfRows();
    int numOfColumns();

    int get(int i, int j);
    void set(int i, int j, int v);

    Matrix add(Matrix other);
    Matrix multiply(Matrix other);

    void print();
}
```

```
abstract class AbstractMatrix implements Matrix { ... }
```

```
class DenseMatrix(...) extends AbstractMatrix { ... }
```

```
class SparseMatrix(...) extends AbstractMatrix { ... }
```

```
class DiagonalMatrix(...) extends AbstractMatrix { ... }
```

```
class UnityMatrix(...) implements Matrix { ... }
```

it's time to... RECAP

a class defines a type & provides an implementation for it

a subclass defines a subtype & provides an implementation for it

a type is just a specification

an abstract class defines a type & provides a partial implementation for it

in scala, a trait allows you to define types without an implementation

in java, an interface forces you to define types without an implementation

in swift, the notion of protocols is similar to traits

in python, this notion has no equivalent due to the absence of static typing

a class can inherit from only one class but from multiple traits / interfaces / protocol