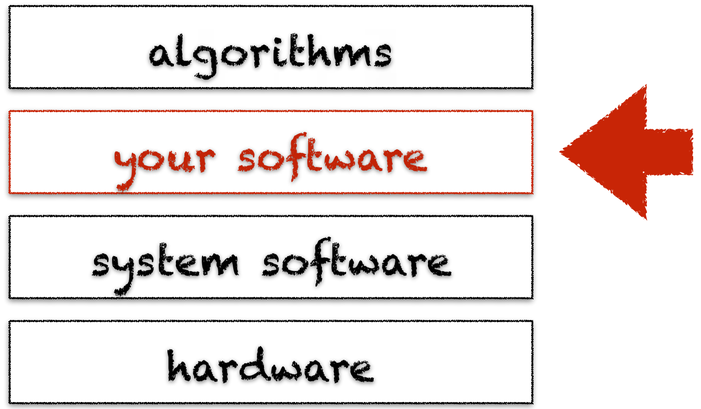
abstract classes & types

learning objectives

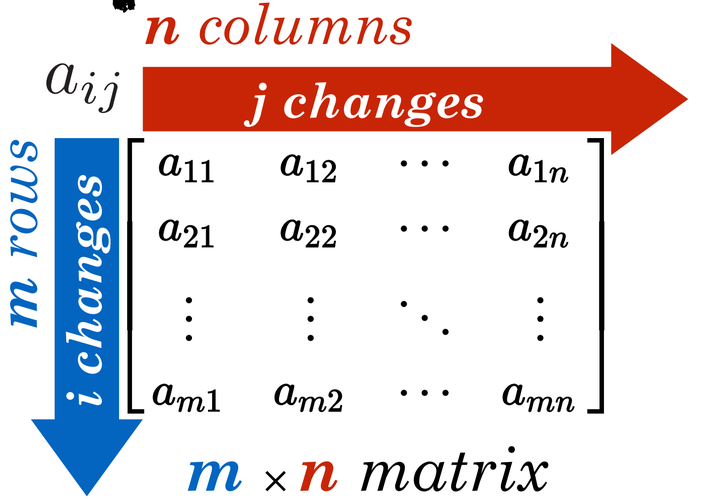


- learn how to define and use abstract classes
- learn how to define types without implementation
- learn about multiple inheritance of types

a mathematical example

representing matrices

a rectangular array of elements arranged in rows and columns



i designates the row
j designates the column

examples

$$\begin{bmatrix} 4 \\ 1 \\ 8 \end{bmatrix} \quad \begin{bmatrix} 3 & 7 & 2 \end{bmatrix} \quad \begin{bmatrix} 9 & 13 & 5 \\ 1 & 11 & 7 \\ 2 & 6 & 3 \end{bmatrix}$$

3×1 *matrix* 1×3 *matrix* 3×3 *matrix*

---

matrices are used in many branches of physics, math, computer graphics, etc.

linear equation

$$\begin{aligned} a_{11}x_1 + a_{12}x_2 &= b_1 \\ a_{21}x_1 + a_{22}x_2 &= b_2 \end{aligned} \quad A\vec{x} = \vec{b} \text{ where } A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \quad \vec{x} = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \quad \vec{b} = \begin{bmatrix} b_1 \\ b_2 \end{bmatrix}$$

representing matrices

sparse matrix

one with most of its elements equal to zero

dense matrix

one with most of its elements not equal to zero

square matrix

one with equal number of rows and columns

diagonal matrix

one with all off-diagonal elements equal to zero

$$d_{i,j} = 0 \text{ if } i \neq j \; \forall i, j \in \{1, 2, \ldots, n\}$$

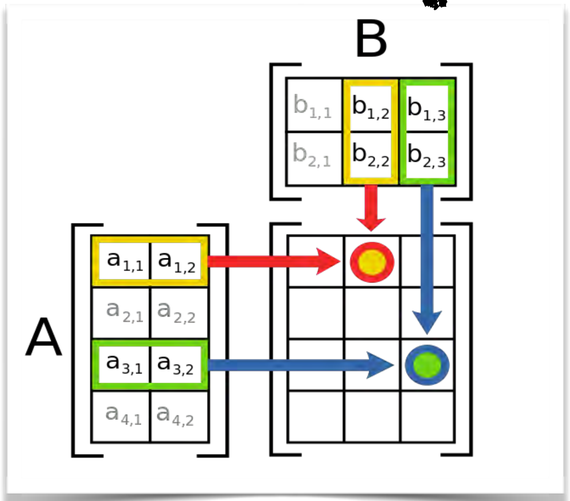
addition

$$\mathbf{A} + \mathbf{B} = \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{bmatrix} + \begin{bmatrix} b_{11} & b_{12} & \cdots & b_{1n} \\ b_{21} & b_{22} & \cdots & b_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ b_{m1} & b_{m2} & \cdots & b_{mn} \end{bmatrix} = \begin{bmatrix} a_{11} + b_{11} & a_{12} + b_{12} & \cdots & a_{1n} + b_{1n} \\ a_{21} + b_{21} & a_{22} + b_{22} & \cdots & a_{2n} + b_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{m1} + b_{m1} & a_{m2} + b_{m2} & \cdots & a_{mn} + b_{mn} \end{bmatrix}$$

$$\begin{bmatrix} 1 & 3 \\ 1 & 0 \\ 1 & 2 \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 7 & 5 \\ 2 & 1 \end{bmatrix} = \begin{bmatrix} 1+0 & 3+0 \\ 1+7 & 0+5 \\ 1+2 & 2+1 \end{bmatrix}$$

$\mathbf{A}$ and $\mathbf{B}$ must have the same number of rows and columns

representing matrices multiplication



$$\mathbf{A} = \begin{pmatrix} A_{11} & A_{12} & \cdots & A_{1m} \\ A_{21} & A_{22} & \cdots & A_{2m} \\ \vdots & \vdots & \ddots & \vdots \\ A_{n1} & A_{n2} & \cdots & A_{nm} \end{pmatrix} \quad \mathbf{B} = \begin{pmatrix} B_{11} & B_{12} & \cdots & B_{1p} \\ B_{21} & B_{22} & \cdots & B_{2p} \\ \vdots & \vdots & \ddots & \vdots \\ B_{m1} & B_{m2} & \cdots & B_{mp} \end{pmatrix} \quad \mathbf{AB} = \begin{pmatrix} (\mathbf{AB})_{11} & (\mathbf{AB})_{12} & \cdots & (\mathbf{AB})_{1p} \\ (\mathbf{AB})_{21} & (\mathbf{AB})_{22} & \cdots & (\mathbf{AB})_{2p} \\ \vdots & \vdots & \ddots & \vdots \\ (\mathbf{AB})_{n1} & (\mathbf{AB})_{n2} & \cdots & (\mathbf{AB})_{np} \end{pmatrix} \quad (\mathbf{AB})_{ij} = \sum_{k=1}^{m} A_{ik} B_{kj}$$

$$\mathbf{A} = \begin{pmatrix} a & b & c \\ p & q & r \\ u & v & w \end{pmatrix} \quad \mathbf{B} = \begin{pmatrix} \alpha & \beta & \gamma \\ \lambda & \mu & \nu \\ \rho & \sigma & \tau \end{pmatrix}$$

$$\mathbf{AB} = \begin{pmatrix} a & b & c \\ p & q & r \\ u & v & w \end{pmatrix} \begin{pmatrix} \alpha & \beta & \gamma \\ \lambda & \mu & \nu \\ \rho & \sigma & \tau \end{pmatrix} = \begin{pmatrix} a\alpha + b\lambda + c\rho & a\beta + b\mu + c\sigma & a\gamma + b\nu + c\tau \\ p\alpha + q\lambda + r\rho & p\beta + q\mu + r\sigma & p\gamma + q\nu + r\tau \\ u\alpha + v\lambda + w\rho & u\beta + v\mu + w\sigma & u\gamma + v\nu + w\tau \end{pmatrix}$$

$$\mathbf{BA} = \begin{pmatrix} \alpha & \beta & \gamma \\ \lambda & \mu & \nu \\ \rho & \sigma & \tau \end{pmatrix} \begin{pmatrix} a & b & c \\ p & q & r \\ u & v & w \end{pmatrix} = \begin{pmatrix} \alpha a + \beta p + \gamma u & \alpha b + \beta q + \gamma v & \alpha c + \beta r + \gamma w \\ \lambda a + \mu p + \nu u & \lambda b + \mu q + \nu v & \lambda c + \mu r + \nu w \\ \rho a + \sigma p + \tau u & \rho b + \sigma q + \tau v & \rho c + \sigma r + \tau w \end{pmatrix}$$

$$\mathbf{AB} \neq \mathbf{BA}$$

representing matrices

```
public class ArrayMatrix  {
    final private int[][] matrix;
    final int numOfRows;
    final int numOfCols;

    public ArrayMatrix(int rows, int cols) {
        if (rows <= 0 || cols <= 0) {
            throw new IllegalArgumentException("A matrix must have a positive number of rows and columns");
        }
        this.numOfRows = rows;
        this.numOfCols = cols;
        matrix = new int[rows][cols];
    }
    public ArrayMatrix(int rows, int cols, int[][] data) {
        this(rows, cols);
        for (int i = 0; i < rows; i++) {
            for (int j = 0; j < cols; j++) {
                this.set(i, j, data[i][j]);
            }
        }
    }
    public int getNumOfRows() { return numOfRows; }
    public int getNumOfCols() { return numOfCols; }
    public int get(int i, int j) { return matrix[i][j]; }
    public void set(int i, int col, int v) { matrix[i][col] = v; }
    public String toString() {
        StringBuilder description = new StringBuilder();
        description.append("\n").append(this.getClass().getSimpleName()).append("\n");
        for (int i = 0; i < numOfRows; i++) {
            description.append("| ");
            for (int j = 0; j < numOfCols; j++) {
                String entry = String.format("% 3d", this.get(i, j));
                description.append(entry).append(" ");
            }
            description.append("|\n");
        }
        return description.toString();
    }
    ⋮
}
```

```
ArrayMatrix m = new ArrayMatrix(3, 3);
System.out.println("m = " + m);

int[][] data = {
   {2, 4, 6},
   {3, 6, 9}
};

m = new ArrayMatrix(2, 3, data);
System.out.println("m = " + m);
```

```
m =
ArrayMatrix
|   0   0   0 |
|   0   0   0 |
|   0   0   0 |

m =
ArrayMatrix
|   2   4   6 |
|   3   6   9 |
```

in the following, we assume that indices go from 0 to $n-1$ rather than 1 to n

representing matrices

```
public class ArrayMatrix  {
    final private int[][] matrix;
    final int numOfRows;
    final int numOfCols;

    public ArrayMatrix(int rows, int cols) {
        if (rows <= 0 || cols <= 0) {
            throw new IllegalArgumentException("A matrix must have a positive number of rows and columns");
        }
        this.numOfRows = rows;
        this.numOfCols = cols;
        matrix = new int[rows][cols];
    }
    public ArrayMatrix(int rows, int cols, int[][] data) {
        this(rows, cols);
        for (int i = 0; i < rows; i++) {
            for (int j = 0; j < cols; j++) {
                this.set(i, j, data[i][j]);
            }
        }
    }
    public int getNumOfRows() { return numOfRows; }
    public int getNumOfCols() { return numOfCols; }
    public int get(int i, int j) { return matrix[i][j]; }
    public void set(int i, int col, int v) { matrix[i][col] = v; }
    public String toString() {
        StringBuilder description = new StringBuilder();
        description.append("\n").append(this.getClass().getSimpleName()).append("\n");
        for (int i = 0; i < numOfRows; i++) {
            description.append("| ");
            for (int j = 0; j < numOfCols; j++) {
                String entry = String.format("% 3d", this.get(i, j));
                description.append(entry).append(" ");
            }
            description.append("|\n");
        }
        return description.toString();
    }
    ⋮
}
```

```
int[][] data = {
   {2, 4, 6},
   {3, 6, 9}
};
ArrayMatrix m = new ArrayMatrix(2, 3, data);
System.out.println("m = " + m);

System.out.println("m(0,0) = " + m.get(0, 0));
m.set(0, 0, 7);
System.out.println("m(0,0) = " + m.get(0, 0));

System.out.println();
System.out.println("m = " + m);
```

```
m =
ArrayMatrix
|   2     4     6   |
|   3     6     9   |

m(0,0) = 2
m(0,0) = 7

m =
ArrayMatrix
|   7     4     6   |
|   3     6     9   |
```

in the following, we assume that indices go from 0 to $n-1$ rather than 1 to n

representing matrices

```java
public class ArrayMatrix  {
    private int[][] matrix;
    final int numOfRows;
    final int numOfCols;
    ⋮
    public ArrayMatrix addTo(ArrayMatrix other) {
        if (this.numOfRows != other.numOfRows || this.numOfCols != other.numOfCols) {
            throw new IllegalArgumentException("Matrices must have the same number of rows and columns when added");
        }

        ArrayMatrix result = new ArrayMatrix(numOfRows, numOfCols);
        for (int i = 0; i < numOfRows; i++) {
            for (int j = 0; j < numOfCols; j++) {
                result.set(i, j, this.get(i, j) + other.get(i, j));
            }
        }

        return result;
    }

    public ArrayMatrix multiplyBy(ArrayMatrix other) {
        if (this.numOfCols != other.getNumOfRows()) {
            throw new IllegalArgumentException("Matrices must have compatible number of rows and columns when multiplied");
        }

        ArrayMatrix result = new ArrayMatrix(this.numOfRows, other.getNumOfCols());
        for (int i = 0; i < result.getNumOfRows(); i++) {
            for (int j = 0; j < result.getNumOfCols(); j++) {
                int entry = 0;
                for (int k = 0; k < this.numOfCols; k++) {
                    entry = entry + this.get(i, k) * other.get(k, j);
                    result.set(i, j, entry);
                }

        }
        return result;
    }
}
```

```java
int[][] data1 = {
    { 1,   2, -4},
    {-1,  -3,  3}
};
int[][] data2 = {
    { 0,   5,  4},
    {-2,  -3,  6},
    { 1,   2,  3}
};
```

```java
ArrayMatrix m1 = new ArrayMatrix(2, 3, data1);
ArrayMatrix m2 = new ArrayMatrix(3, 3, data2);
ArrayMatrix m3 = m2.addTo(m2);
System.out.println("m3 = m2 + m2 = " + m3);

m3 = m1.multiplyBy(m2);
System.out.println("m3 = m1 * m2 = " + m3);
```

```
m3 = m2 + m2 =
ArrayMatrix
|   0    10     8   |
|  -4    -6    12   |
|   2     4     6   |

m3 = m1 * m2 =
ArrayMatrix
|  -8    -9     4   |
|   9    10   -13   |
```

$$(\mathbf{AB})_{ij} = \sum_{k=1}^{m} A_{ik} B_{kj}$$

sparse matrices

```
public class MapMatrix {
    final Map<Index, Integer> matrix;
    final int numOfRows;
    final int numOfCols;

    public MapMatrix(int rows, int cols) {
        if (rows <= 0 || cols <= 0) {
            throw new IllegalArgumentException("A matrix must have a positive number of rows and columns");
        }
        this.numOfRows = rows;
        this.numOfCols = cols;
        matrix = new HashMap();
    }
  public MapMatrix(int rows, int cols, Map<Index, Integer> data) {
        this(rows, cols);
        for (Index index : data.keySet()) {
            if (index.i < rows && index.j < cols) {
                this.set(index.i, index.j, data.get(index));
            }
        }
    }
    public int getNumOfRows() { return numOfRows; }
    public int getNumOfCols() { return numOfCols; }
    public int get(int i, int j) {
        Index index = Index.from(i, j);
        return matrix.containsKey(index) ? matrix.get(index) : 0;
    }
    public void set(int i, int j, int v) { matrix.put(Index.from(i, j), v); }

    public String toString() {
        StringBuilder description = new StringBuilder();
       description.append("\n").append(this.getClass().getSimpleName()).append("\n");
        for (int i = 0; i < numOfRows; i++) {
            description.append("| ");
            for (int j = 0; j < numOfCols; j++) {
                String entry = String.format("% 3d", this.get(i, j));
                description.append(entry).append("   ");
            }
            description.append("|\n");
        }
        return description.toString();
    }
    ⋮
}
```

```
MapMatrix m = new MapMatrix(3, 3);
System.out.println("m = " + m);

Map<Index, Integer> data = new HashMap();
data.put(Index.from(2, 4), 66);
data.put(Index.from(0, 2), 44);

m = new MapMatrix(5, 5, data);
System.out.println("m = " + m);
```

```
m =
MapMatrix
|   0   0   0   |
|   0   0   0   |
|   0   0   0   |

m =
MapMatrix
|   0   0  44   0   0   |
|   0   0   0   0   0   |
|   0   0   0   0  66   |
|   0   0   0   0   0   |
|   0   0   0   0   0   |
```

sparse matrices

```
public class MapMatrix {
    final Map<Index, Integer> matrix;
    final int numOfRows;
    final int numOfCols;

    public MapMatrix(int rows, int cols) {
        if (rows <= 0 || cols <= 0) {
            throw new IllegalArgumentException("A matrix must have a positive number o
        }
        this.numOfRows = rows;
        this.numOfCols = cols;
        matrix = new HashMap();
    }
  public MapMatrix(int rows, int cols, Map<Index, Integer> data) {
        this(rows, cols);
        for (Index index : data.keySet()) {
            if (index.i < rows && index.j < cols) {
                this.set(index.i, index.j, data.get(index));
            }
        }
    }
```

```
MapMatrix m = new MapMatrix(3, 3);
System.out.println("m = " + m);

Map<Index, Integer> data = new HashMap();
data.put(Index.from(2, 4), 66);
data.put(Index.from(0, 2), 44);

m = new MapMatrix(5, 5, data);
System.out.println("m = " + m);
```

```
public class Index {
    public final int i;
    public final int j;
    public Index(int i, int j) {
        this.i = i;
        this.j = j;
    }
    public static Index from(int i, int j) {
        return new Index(i, j);
    }
    public int hashCode() {
        int hash = 7;
        hash = 83 * hash + this.i;
        hash = 83 * hash + this.j;
        return hash;
    }
    ⋮
```

```
    ⋮
    public boolean equals(Object obj) {
        if (this == obj) {
            return true;
        }
        if (obj == null) {
            return false;
        }
        if (getClass() != obj.getClass()) {
            return false;
        }
        final Index other = (Index) obj;
        if (this.i != other.i) {
            return false;
        }
        if (this.j != other.j) {
            return false;
        }
        return true;
    }
}
```

problem

```java
public class MapMatrix {
    final Map<Index, Integer> matrix;
    final int numOfRows;
    final int numOfCols;

    public MapMatrix(int rows, int cols) {
        if (rows <= 0 || cols <= 0) {
            throw new IllegalArgumentException("A matrix must have a positive number of
        }
        this.numOfRows = rows;
        this.numOfCols = cols;
        matrix = new HashMap();
    }
  public MapMatrix(int rows, int cols, Map<Index, Integer> data) {
        this(rows, cols);
        for (Index index : data.keySet()) {
            if (index.i < rows && index.j < cols) {
                this.set(index.i, index.j, data.get(index));
            }
        }
    }
    public int getNumOfRows() { return numOfRows; }
    public int getNumOfCols() { return numOfCols; }
    public int get(int i, int j) {
        Index index = Index.from(i, j);
        return matrix.containsKey(index) ? matrix.get(index) : 0;
    }
    public void set(int i, int j, int v) { matrix.put(Index.from(i, j), v); }

    public String toString() {
        StringBuilder description = new StringBuilder();
       description.append("\n").append(this.getClass().getSimpleName()).append("\n");
        for (int i = 0; i < numOfRows; i++) {
            description.append("| ");
            for (int j = 0; j < numOfCols; j++) {
                String entry = String.format("% 3d", this.get(i, j));
                description.append(entry).append("   ");
            }
            description.append("|\n");
        }
        return description.toString();
    }
    ⋮
}
```

```java
public class ArrayMatrix  {
    private int[][] matrix;
    final int numOfRows;
    final int numOfCols;

    public ArrayMatrix(int rows, int cols) {
        if (rows <= 0 || cols <= 0) {
            throw new IllegalArgumentException("A matrix must have a positive number of rows and c
        }
        this.numOfRows = rows;
        this.numOfCols = cols;
        matrix = new int[rows][cols];
    }
    public ArrayMatrix(int rows, int cols, int[][] data) {
        this(rows, cols);
        for (int i = 0; i < rows; i++) {
            for (int j = 0; j < cols; j++) {
                this.set(i, j, data[i][j]);
            }
        }
    }
    public int getNumOfRows() { return numOfRows; }
    public int getNumOfCols() { return numOfCols; }
    public int get(int i, int j) { return matrix[i][j]; }


    public void set(int i, int col, int v) { matrix[i][col] = v; }

    public String toString() {
        StringBuilder description = new StringBuilder();
        description.append("\n").append(this.getClass().getSimpleName()).append("\n");
        for (int i = 0; i < numOfRows; i++) {
            description.append("| ");
            for (int j = 0; j < numOfCols; j++) {
                String entry = String.format("% 3d", this.get(i, j));
                description.append(entry).append(" ");
            }
            description.append("|\n");
        }
        return description.toString();
    }
    ⋮
```

code duplication... again!

problem

can we blend array and map matrices?

```java
public ArrayMatrix multiplyBy(ArrayMatrix other) {
    if (this.numOfCols != other.getNumOfRows()) {
        throw new IllegalArgumentException("Matrices must have compatible number of rows and columns when multiplied");
    }
    ArrayMatrix result = new ArrayMatrix(this.numOfRows, other.getNumOfCols());
    for (int i = 0; i < result.getNumOfRows(); i++) {
        for (int j = 0; j < result.getNumOfCols(); j++) {
            int entry = 0;
            for (int k = 0; k < this.numOfCols; k++) {
                entry = entry + this.get(i, k) * other.get(k, j);
                result.set(i, j, entry);
            }

    }
    return result;
}
```

```java
public MapMatrix multiplyBy(MapMatrix other) {
    if (this.numOfCols != other.getNumOfRows()) {
        throw new IllegalArgumentException("Matrices must have compatible number of rows and columns when multiplied");
    }
    MapMatrix result = new MapMatrix(this.numOfRows, other.getNumOfCols());
    for (int i = 0; i < result.getNumOfRows(); i++) {
        for (int j = 0; j < result.getNumOfCols(); j++) {
            int entry = 0;
            for (int k = 0; k < this.numOfCols; k++) {
                entry = entry + this.get(i, k) * other.get(k, j);
                result.set(i, j, entry);
            }

    }
    return result;
}
```

incompatible types

representing matrices

```
class Matrix(private val rows: Int, private val cols: Int) {
 if (rows <= 0)
    throw new IllegalArgumentException("A matrix must have a positive number of rows")
  if (cols <= 0)
    throw new IllegalArgumentException("A matrix must have a positive number of columns")

  private var matrix = Array.ofDim[Int](rows, cols)

  def this(rows: Int, cols: Int, mtx: Array[Array[Int]]) {
    this(rows,cols)
    for (i <- 0 to rows - 1)
      mtx(i).copyToArray(matrix(i))
  }

  def numOfRows: Int = this.rows
  def numOfCols: Int = this.cols

  def apply(i: Int, j: Int): Int = matrix(i)(j)
  def update(i: Int, j: Int, v: Int) = matrix(i)(j) = v


  def print: Unit = {
    println(this.getClass.getSimpleName)
    for (i <- 0 to this.numOfRows - 1) {
      Console.print("|  ")
      for (j <- 0 to this.numOfCols - 1) {
        Console.print(s"${this(i,j)}  ")
      }
      println("|")
    }
  }
 ⋮
```

```
var m = new Matrix(3,3)
m.print()

val mtx : Array[Array[Int]] = Array( Array(2, 4, 6) , Array(3, 6, 9))

m = new Matrix(2,3,mtx)
m.print()
```

```
Matrix
|  0  0  0  |
|  0  0  0  |
|  0  0  0  |
Matrix
|  2  4  6  |
|  3  6  9  |
```

in the following, we assume that indices go from 0 to $n–1$ rather than 1 to n

representing matrices

```
class Matrix(private val rows: Int, private val cols: Int) {
 if (rows <= 0)
    throw new IllegalArgumentException("A matrix must have a positive number of rows")
  if (cols <= 0)
    throw new IllegalArgumentException("A matrix must have a positive number of columns")

  private var matrix = Array.ofDim[Int](rows, cols)

  def this(rows: Int, cols: Int, mtx: Array[Array[Int]]) {
    this(rows,cols)
    for (i <- 0 to rows - 1)
      mtx(i).copyToArray(matrix(i))
  }

  def numOfRows: Int = this.rows
  def numOfCols: Int = this.cols

  def apply(i: Int, j: Int): Int = matrix(i)(j)
  def update(i: Int, j: Int, v: Int) = matrix(i)(j) = v

  def print: Unit = {
    println(this.getClass.getSimpleName)
    for (i <- 0 to this.numOfRows - 1) {
      Console.print("|  ")
      for (j <- 0 to this.numOfCols - 1) {
        Console.print(s"${this(i,j)}  ")
      }
      println("|")
    }
  }
 ⋮
```

```
val mtx : Array[Array[Int]] = Array( Array(2, 4, 6) , Array(3, 6, 9))
m = new Matrix(2,3,mtx)
m.print()

println(s"m(0,0) = ${m(0,0)}"); m(0,0) = 7; println(s"m(0,0) = ${m(0,0)}")
m.print()
```

```
Matrix
|  2  4  6  |
|  3  6  9  |
m(0,0) = 2
m(0,0) = 7
Matrix
|  7  4  6  |
|  3  6  9  |
```

apply(…) ⇔ get(…)
update(…) ⇔ set(…)

representing matrices

```
class Matrix(private val rows: Int, private val cols: Int) {
  ⋮
  def +(other: Matrix): Matrix = {
    if (this.numOfRows != other.numOfRows || this.numOfCols != other.numOfCols)
      throw new IllegalArgumentException("Matrices must have the same number of rows" + " and columns when added")

    val result = new Matrix(this.numOfRows, this.numOfCols)

    for (i <- 0 to this.numOfRows - 1; j <- 0 to this.numOfCols - 1)
      result(i,j) = this (i, j) + other(i, j)
    return result
  }

  def *(other: Matrix): Matrix = {
    if (this.numOfCols != other.numOfRows)
      throw new IllegalArgumentException("The number of columns in the first matrix must be" +
                          "equal to the number of rows of the second matrix when multiplied")

    val result = new Matrix(this.numOfRows, other.numOfCols)

    for (i <- 0 to result.numOfRows - 1; j <- 0 to result.numOfCols - 1) {
      var entry: Int = 0
      for (k <- 0 to this.numOfCols - 1)
        entry = entry + this (i, k) * other(k, j)
      result(i,j) = entry
    }
    return result
  }
}
```

$$(\mathbf{AB})_{ij} = \sum_{k=1}^{m} A_{ik} B_{kj}$$

sparse matrices

```
class SparseMatrix(private val rows: Int, private val cols: Int) {
 if (rows <= 0)
    throw new IllegalArgumentException("A matrix must have a positive number of rows")
  if (cols <= 0)
    throw new IllegalArgumentException("A matrix must have a positive number of columns")

  private var matrix = scala.collection.mutable.Map[(Int,Int),Int]()

  def this(rows: Int, cols: Int, mtx: Map[(Int,Int),Int]) {
    this(rows,cols)
    matrix = collection.mutable.Map(mtx.toSeq: _*)
  }

  def numOfRows: Int = this.rows
  def numOfCols: Int = this.cols

  def apply(i: Int, j: Int): Int = matrix.getOrElse((i,j),0)
  def update(i: Int, j: Int, v: Int): Unit = this.matrix((i,j)) = v


  def print: Unit = {
    println(this.getClass.getSimpleName)
    for (i <- 0 to this.numOfRows - 1) {
      Console.print("|  ")
      for (j <- 0 to this.numOfCols - 1) {
        Console.print(s"${this(i,j)}  ")
      }
      println("|")
    }
  }
 ⋮
}
```

```
var m = new SparseMatrix(3,3)
m.print()
```

```
SparseMatrix
|  0  0  0  |
|  0  0  0  |
|  0  0  0  |
```

```
val mtx = scala.collection.immutable.Map((6,6) -> 6, (3,3) -> 3)
m = new SparseMatrix(10,10, mtx)
m.print()
```

```
SparseMatrix
|  0  0  0  0  0  0  0  0  0  0  |
|  0  0  0  0  0  0  0  0  0  0  |
|  0  0  0  0  0  0  0  0  0  0  |
|  0  0  0  3  0  0  0  0  0  0  |
|  0  0  0  0  0  0  0  0  0  0  |
|  0  0  0  0  0  0  0  0  0  0  |
|  0  0  0  0  0  0  6  0  0  0  |
|  0  0  0  0  0  0  0  0  0  0  |
|  0  0  0  0  0  0  0  0  0  0  |
|  0  0  0  0  0  0  0  0  0  0  |
```

problem

```scala
class Matrix(private val rows: Int, private val cols: Int) {
 if (rows <= 0)
    throw new IllegalArgumentException("A matrix must have a positive num
  if (cols <= 0)
    throw new IllegalArgumentException("A matrix must have a positive num

  private var matrix = Array.ofDim[Int](rows, cols)

  def this(rows: Int, cols: Int, mtx: Array[Array[Int]]) {
    this(rows,cols)
    for (i <- 0 to rows - 1)
      mtx(i).copyToArray(matrix(i))
  }

  def numOfRows: Int = this.rows
  def numOfCols: Int = this.cols

  def apply(i: Int, j: Int): Int = matrix(i)(j)
  def update(i: Int, j: Int, v: Int) = matrix(i)(j) = v

  def print: Unit = {
    println(this.getClass.getSimpleName)
    for (i <- 0 to this.numOfRows - 1) {
      Console.print("|  ")
      for (j <- 0 to this.numOfCols - 1) {
        Console.print(s"${this(i,j)}  ")
      }
      println("|")
    }
  }

  ...
```

```scala
class SparseMatrix(private val rows: Int, private val cols: Int) {
 if (rows <= 0)
    throw new IllegalArgumentException("A matrix must have a positive number of rows")
  if (cols <= 0)
    throw new IllegalArgumentException("A matrix must have a positive number of columns")

  private var matrix = scala.collection.mutable.Map[(Int,Int),Int]()

  def this(rows: Int, cols: Int, mtx: Map[(Int,Int),Int]) {
    this(rows,cols)
    matrix = collection.mutable.Map(mtx.toSeq: _*)
  }

  def numOfRows: Int = this.rows
  def numOfCols: Int = this.cols

  def apply(i: Int, j: Int): Int = matrix.getOrElse((i,j),0)
  def update(i: Int, j: Int, v: Int): Unit = this.matrix((i,j)) = v

  def print: Unit = {
    println(this.getClass.getSimpleName)
    for (i <- 0 to this.numOfRows - 1) {
      Console.print("|  ")
      for (j <- 0 to this.numOfCols - 1) {
        Console.print(s"${this(i,j)}  ")
      }
      println("|")
    }
  }

  ...
```

code duplication ... again!

problem

can we blend dense and sparse matrices?

```
def *(other: Matrix): Matrix = {
   if (this.numOfCols != other.numOfRows)
     throw new IllegalArgumentException("The number of columns in the first matrix must be" +
                          "equal to the number of rows of the second matrix when multiplied")

   val result = new Matrix(this.numOfRows, other.numOfCols)

   for (i <- 0 to result.numOfRows - 1; j <- 0 to result.numOfCols - 1) {
     var entry: Int = 0
     for (k <- 0 to this.numOfCols - 1)
       entry = entry + this (i, k) * other(k, j)
     result(i,j) = entry
   }
   return result
 }
```

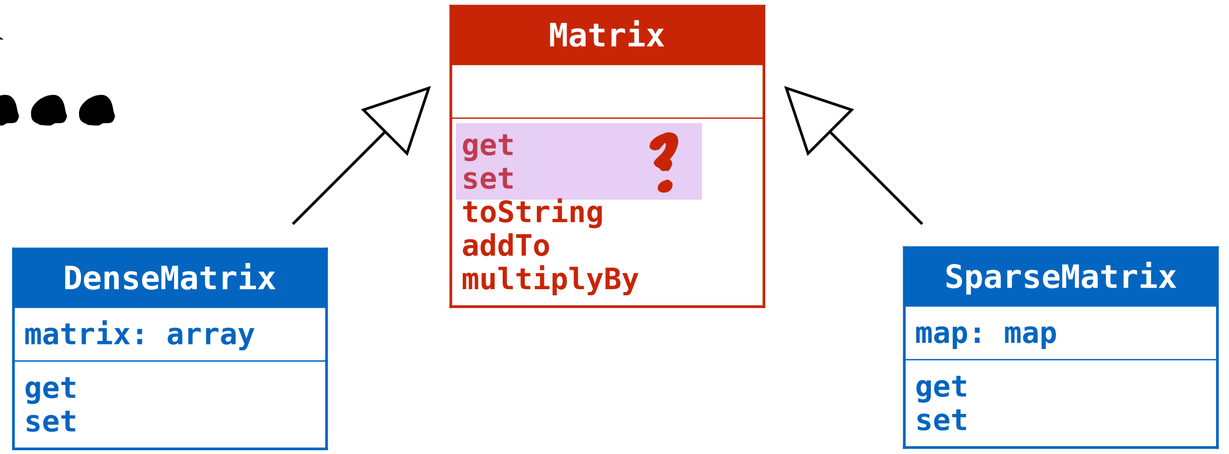
```
def *(other: SparseMatrix): SparseMatrix = {
   if (this.numOfCols != other.numOfRows)
     throw new IllegalArgumentException("The number of columns in the first matrix must be" +
                          "equal to the number of rows of the second matrix when multiplied")

   val result = new SparseMatrix(this.numOfRows, other.numOfCols)

   for (i <- 0 to result.numOfRows - 1; j <- 0 to result.numOfCols - 1) {
     var entry: Int = 0
     for (k <- 0 to this.numOfCols - 1)
       entry = entry + this (i, k) * other(k, j)
     result(i,j) = entry
   }
   return result
 }
```

❌ incompatible types

solution

inheritance?

yes, but...



the superclass delegates the internal representation to its subclasses

so methods get and set have no default or shared implementation

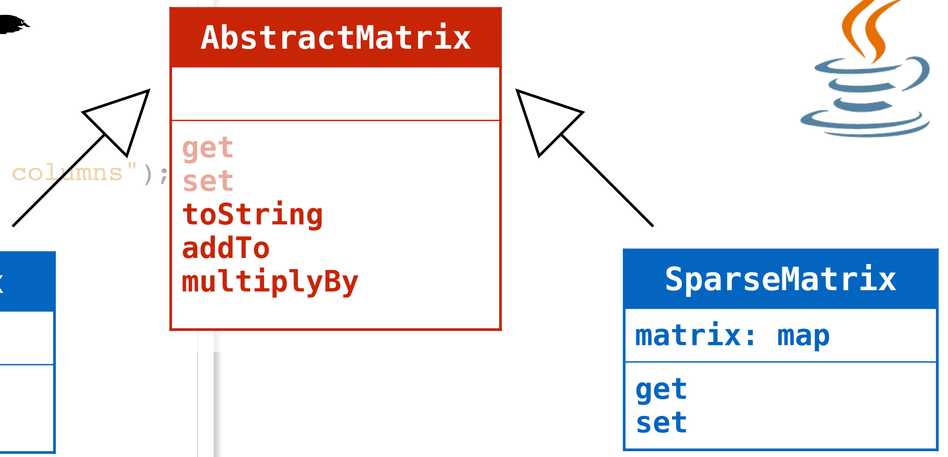
these methods are abstract in the superclass

solution

abstract class!

abstract class

```
public abstract class AbstractMatrix  {
    final int numOfRows;
    final int numOfCols;

    public AbstractMatrix(int rows, int cols) {
        if (rows <= 0 || cols <= 0) {
            throw new IllegalArgumentException("A matrix must have a positive number of rows and columns");
        }
        this.numOfRows = rows;
        this.numOfCols = cols;
    }
    public abstract int get(int i, int j);
    public abstract void set(int i, int j, int v);

    public int getNumOfRows() { return numOfRows; }
    public int getNumOfCols() { return numOfCols; }

    final protected void init(int[][] data) {
        for (int i = 0; i < data.length; i++) {
            for (int j = 0; j < data[i].length; j++) {
                this.set(i, j, data[i][j]);
            }
        }

    }
    final protected void init(Map<Index, Integer> data) {
        for (Index index : data.keySet()) {
            if (index.i < numOfRows && index.j < numOfRows) {
                this.set(index.i, index.j, data.get(index));
            }
        }
    }
    public String toString() {
        StringBuilder description = new StringBuilder();

        description.append("\n").append(this.getClass().getSimp
        for (int i = 0; i < numOfRows; i++) {
            description.append("| ");
            for (int j = 0; j < numOfCols; j++) {
                description.append(this.get(i, j)).append(" ");
            }
            description.append("|\n");
        }
        return description.toString();
    }
    ⋮
}
```

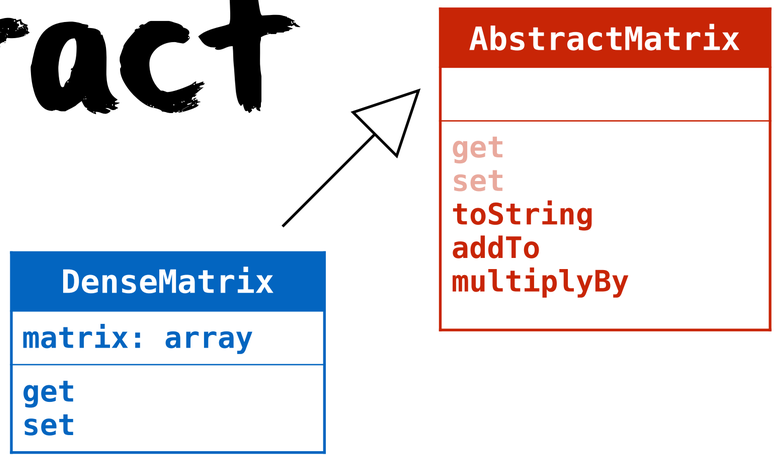



```
    ⋮
    public AbstractMatrix addTo(AbstractMatrix other) {
        if (this.numOfRows != other.numOfRows || this.numOfCols != other.numOfCols) {
            throw new IllegalArgumentException("Matrices must have the same number of rows a
        }

        DenseMatrix result = new DenseMatrix(numOfRows, numOfCols);
        for (int i = 0; i < numOfRows; i++) {
            for (int j = 0; j < numOfCols; j++) {

                result.set(i, j, this.get(i, j) + other.get(i, j));
            }
        }
        return result;
    }
    ⋮
}
```

why use DenseMatrix, not SparseMatrix?

could we come with a better scheme?

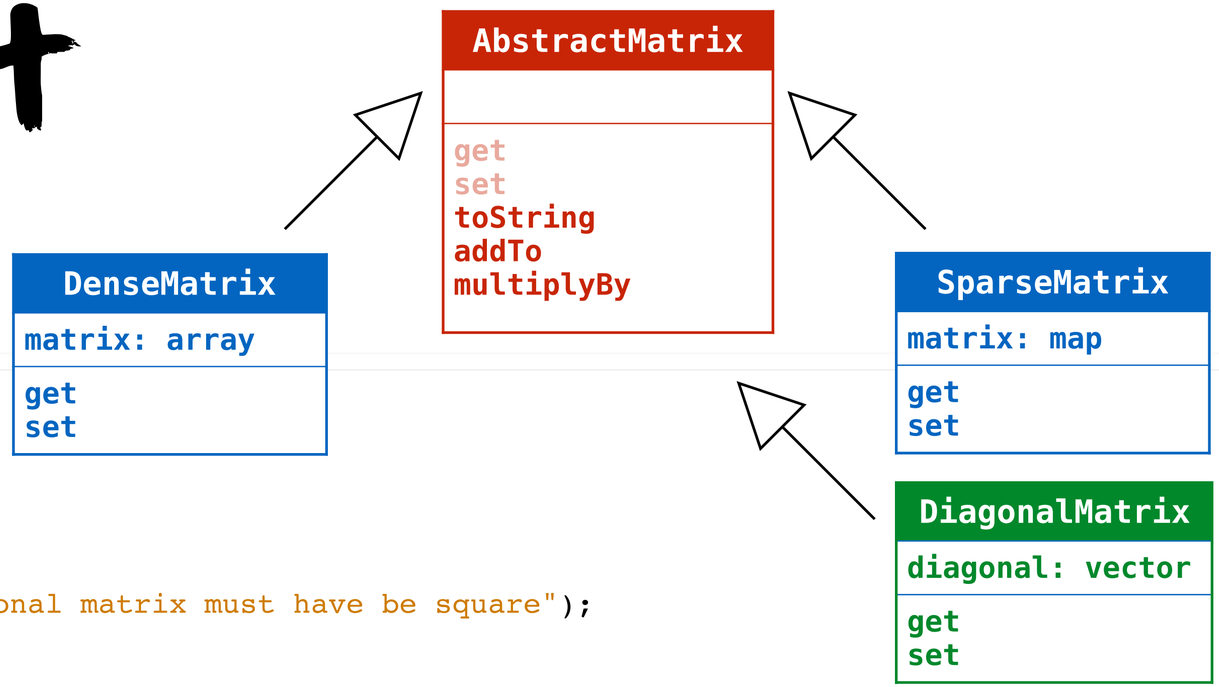
abstract class



```java
public class SparseMatrix extends AbstractMatrix {
    final Map<Index, Integer> matrix;

    public SparseMatrix(int rows, int cols) {
        super(rows, cols);
        matrix = new HashMap();
    }
    public SparseMatrix(int rows, int cols, int[][] data) {
        this(rows, cols);
        init(data);
    }
    public SparseMatrix(int rows, int cols, Map<Index, Integer> data) {
        this(rows, cols);
        init(data);
    }
    public int get(int i, int j) {
        Index index = Index.from(i, j);
        if (matrix.containsKey(index)) {
            return matrix.get(index);
        } else {
            return 0;
        }
    }
    public void set(int i, int j, int v) {
        matrix.put(Index.from(i, j), v);
    }
}
```

```java
public class DenseMatrix extends AbstractMatrix {
    private int[][] matrix;

    public DenseMatrix(int rows, int cols) {
        super(rows, cols);
        matrix = new int[rows][cols];
    }
    public DenseMatrix(int rows, int cols, int[][] data) {
        this(rows, cols);
        init(data);
    }
    public DenseMatrix(int rows, int cols, Map<Index, Integer> data) {
        this(rows, cols);
        init(data);
    }
    public int get(int i, int j) {
        return matrix[i][j];
    }
    public void set(int i, int j, int v) {
        matrix[i][j] = v;
    }
}
```

abstract class




```
public class DiagonalMatrix extends AbstractMatrix {
    private int[] diagonal;

    public DiagonalMatrix(int rows, int cols) {
        super(rows, cols);
        if (rows != cols) {
            throw new IllegalArgumentException("A diagonal matrix must have be square");
        }
        diagonal = new int[rows];
    }
    public DiagonalMatrix(int rows, int cols, int[] diagonal) {
        this(rows, cols);
        int limit = Math.min(rows, diagonal.length);
        for (int i = 0; i < limit; i++) {
            this.diagonal[i] = diagonal[i];
        }
    }
    public int get(int i, int j) {
        return (i == j) ? diagonal[i] : 0;
    }
    public void set(int i, int j, int v) {
        if (i == j) {
             diagonal[i] = v;
        } else {
            throw new IllegalArgumentException("A diagonal matrix should only" + " have zeros outside its diagonal");
        }
    }
}
```

what if all methods could be abstract?

identity matrix

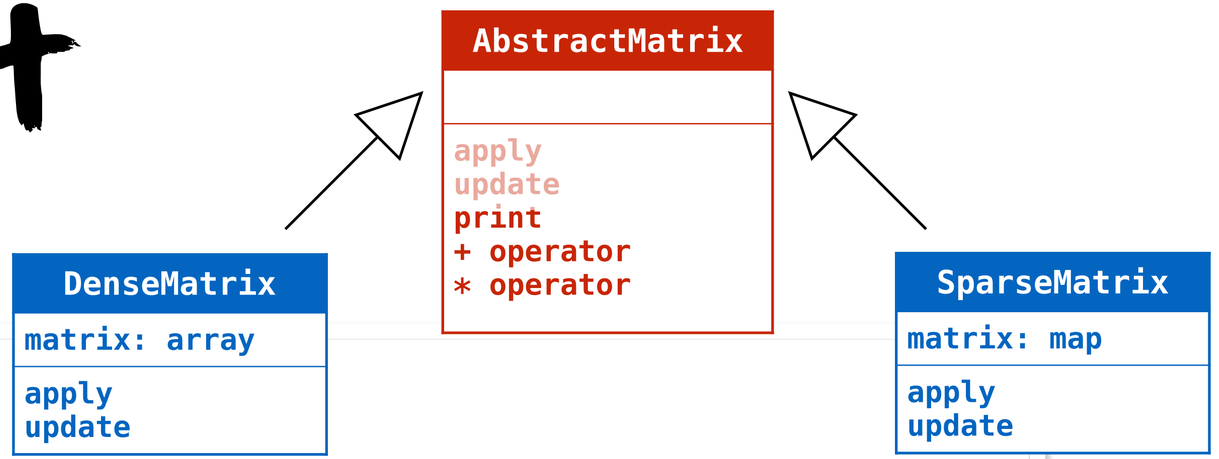
$$A \times I_n = I_n \times A = A$$

$$\begin{bmatrix} 1 & 0 & 0 & \cdots & 0 \\ 0 & 1 & 0 & \cdots & 0 \\ 0 & 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \cdots & 1 \end{bmatrix} = I_n$$

```
public class UnityMatrix extends AbstractMatrix {
   public UnityMatrix(int size) {
        super(size, size);
    }
    public int get(int i, int j) { return (i == j) ? 1 : 0; }
    public void set(int i, int j, int v) { throw new UnsupportedOperationException("Not supported: a unity matrix is constant!"); }

    public AbstractMatrix addTo(AbstractMatrix other) {
        AbstractMatrix result = other.clone();
        for (int i = 0; i < numOfRows; i++) {
            result.set(i, i, other.get(i, i) + 1);
        }
        return result;
    }
    public AbstractMatrix multiplyBy(AbstractMatrix other) {
        return other.clone();
    }
}
```

we have to add a clone method to all classes
(clone = deep copy)

abstract class

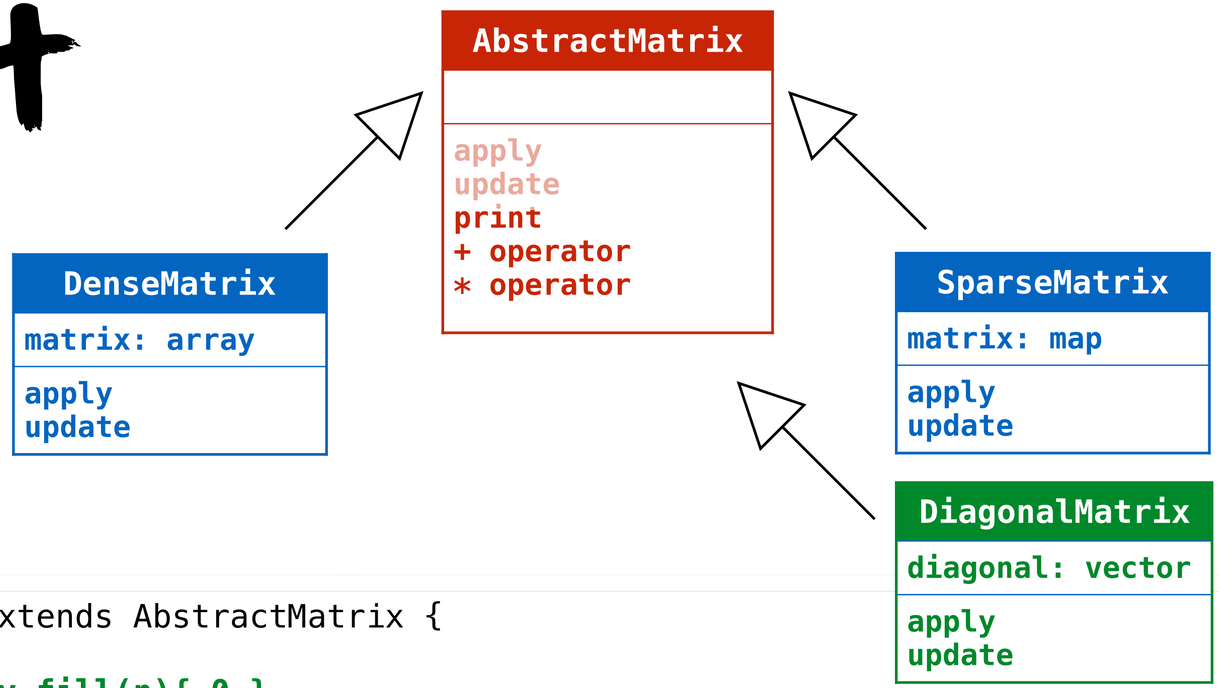



```
abstract class AbstractMatrix {
  def apply(i: Int, j: Int): Int
  def update(i: Int, j: Int, v: Int)

  def +(other: AbstractMatrix): AbstractMatrix = {
    if (this.numOfRows != other.numOfRows || this.numOfCols != other.numOfCols)
      throw new IllegalArgumentException("Matrices must have the same number of rows and columns when added")

    val result = new DenseMatrix(this.numOfRows, this.numOfCols)
    for (i <- 0 to this.numOfRows - 1; j <- 0 to this.numOfCols - 1)
      result(i,j) = this (i, j) + other(i, j)
    return result
  }

  def *(other: AbstractMatrix): AbstractMatrix = {
    if (this.numOfCols != other.numOfRows)
      throw new IllegalArgumentException("The number of columns in the first matrix must be equal" +
                                         " to the number of rows of the second matrix when multiplied")

    val result = new DenseMatrix(this.numOfRows, other.numOfCols)
    for (i <- 0 to result.numOfRows - 1; j <- 0 to result.numOfCols - 1) {
      var entry: Int = 0
      for (k <- 0 to this.numOfCols - 1)
        entry = entry + this (i, k) * other(k, j)
      result(i,j) = entry
    }
    return result
  }
  ⋮
}
```

why use DenseMatrix, not SparseMatrix?

could we come with a better scheme?

abstract class




```
class DiagonalMatrix(private val n: Int) extends AbstractMatrix {

  private var diagonal : Array[Int] = Array.fill(n){ 0 }

  def this(n: Int, diag: Array[Int]) {
    this(n)
    this.diagonal = diag
  }

  def apply(i: Int, j: Int): Int = if (i != j) 0 else diagonal(i)

  def update(i: Int, j: Int, v: Int): Unit = {
    if (i == j)
      this.diagonal(i) = v
    else (i != j && v != 0)
      throw new IllegalArgumentException("A diagonal matrix should only" + " have zeros outside its diagonal")
  }
}
```

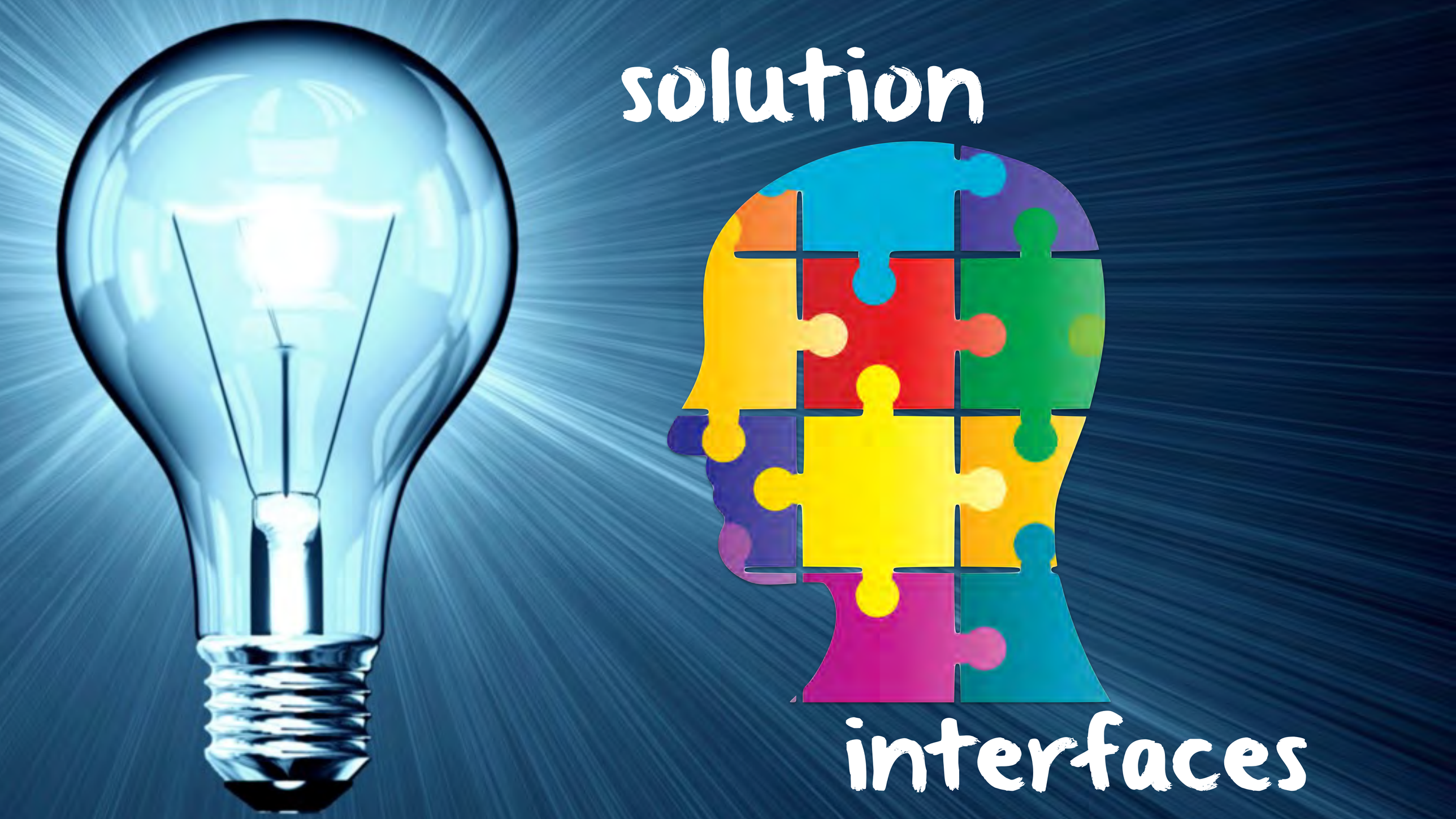
what if all methods could be abstract?

identity matrix

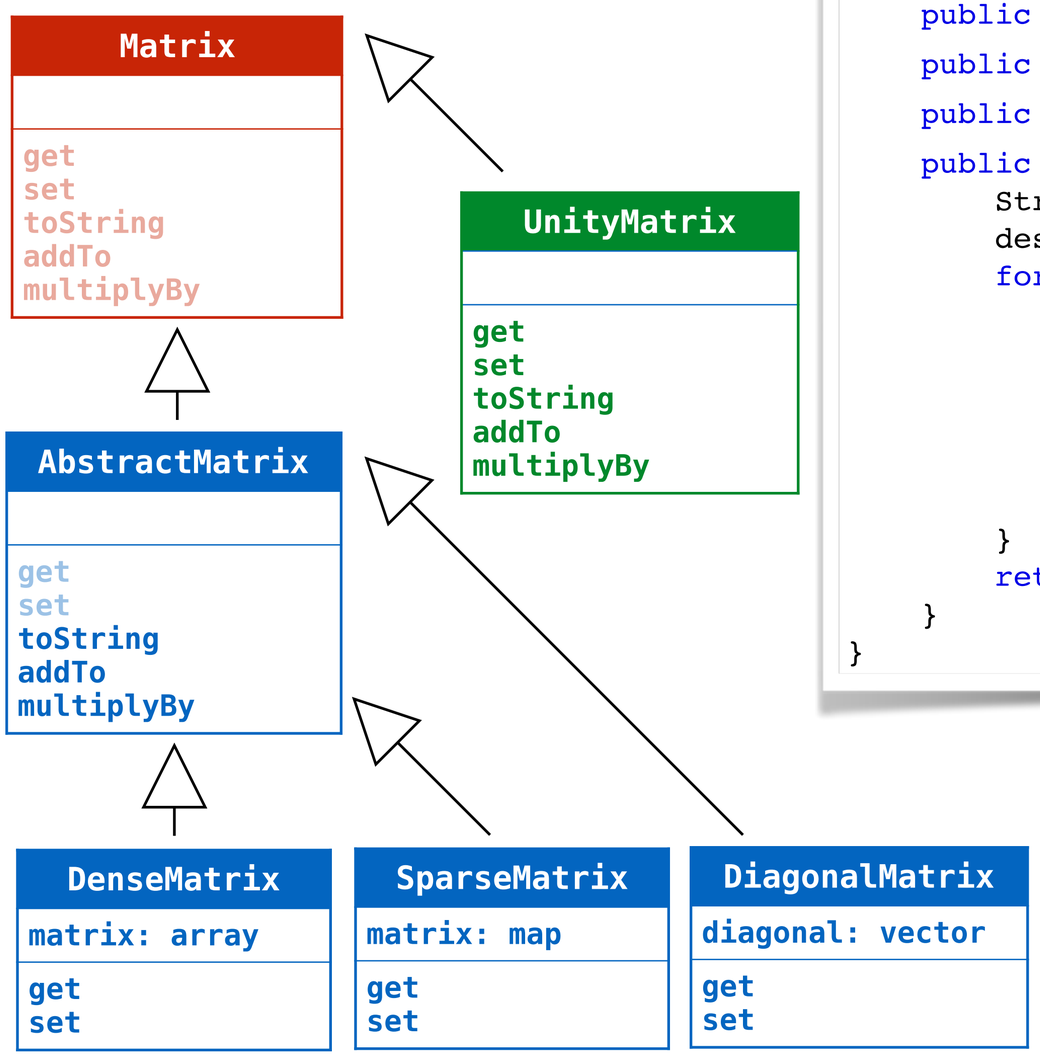
$$A \times I_n = I_n \times A = A$$

$$\begin{bmatrix} 1 & 0 & 0 & \cdots & 0 \\ 0 & 1 & 0 & \cdots & 0 \\ 0 & 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \cdots & 1 \end{bmatrix} = I_n$$

```
class IdentityMatrix(private val n: Int) extends AbstractMatrix {
  def apply(i: Int, j: Int): Int = if (i == j) 1 else 0
  def update(i: Int, j: Int, v: Int): Unit = {
    throw new IllegalArgumentException("Not supported: a unity matrix is constant!")
  }

  override def *(other: Matrix): Matrix = other.duplicate

  override def +(other: Matrix): Matrix = {
    val result = other.duplicate
    for (i <- 0 to n - 1)
      result(i,i) = result(i,i) + 1
    return result
  }
}
```


solution
interfaces

interface




```java
public interface Matrix {
    public Matrix addTo(Matrix other);
    public Matrix multiplyBy(Matrix other);
    public int get(int i, int j);
    public void set(int i, int j, int v);
    public int getNumOfRows();
    public int getNumOfCols();
    public Matrix clone();
    public static String toString(Matrix matrix) {
        StringBuilder description = new StringBuilder();
        description.append("\n").append(matrix.getClass().getSimpleName()).append("\n");
        for (int i = 0; i < matrix.getNumOfRows(); i++) {
            description.append("| ");
            for (int j = 0; j < matrix.getNumOfCols(); j++) {
                String entry = String.format("% 3d", matrix.get(i, j));
                description.append(entry).append(" ");
            }
            description.append("|\n");
        }
        return description.toString();
    }
}
```

```java
abstract class AbstractMatrix implements Matrix { … }
```

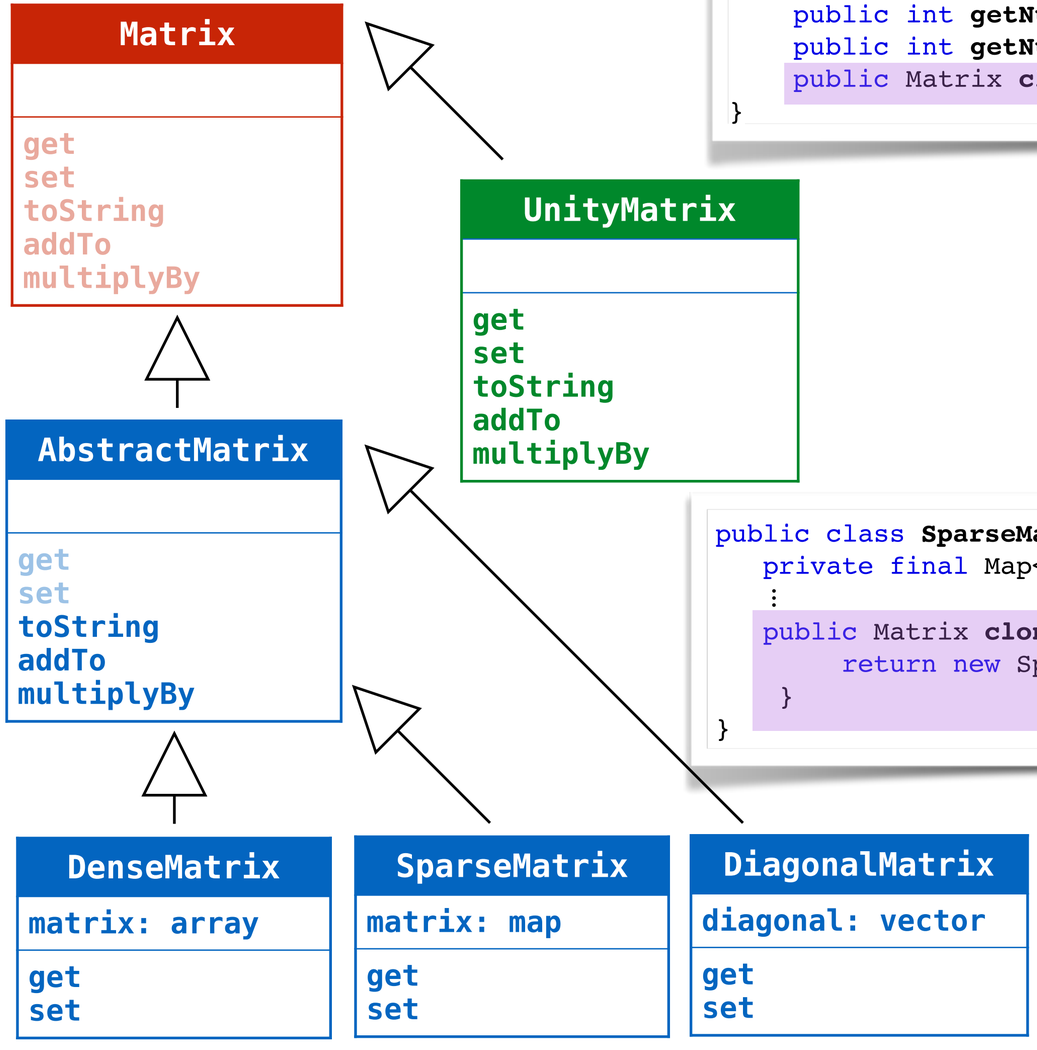
```java
class DenseMatrix extends AbstractMatrix { … }
```

```java
class SparseMatrix extends AbstractMatrix { … }
```

```java
class DiagonalMatrix extends AbstractMatrix { … }
```

```java
class UnityMatrix implements Matrix { … }
```

interface




```java
public class UnityMatrix extends AbstractMatrix {
    int size;
    public UnityMatrix(int size) {
        if (size <= 0) {
            throw new IllegalArgumentException("A matrix must have a positive number of rows and columns");
        }
        this.size = size;
    }
    public String toString()  { return Matrix.toString(this); }
    public int getNumOfRows() { return size; }
    public int getNumOfCols() { return size; }
    public Matrix clone()     { return this; }
}
```

```java
public abstract class AbstractMatrix implements Matrix {
    private final int numOfRows;
    private final int numOfCols;

    final protected void init(int[][] data) {
    final protected void init(Map<Index, Integer> data) { … }
    public Matrix addTo(Matrix other) { … }
    public Matrix multiplyBy(Matrix other) { … }
        if (this.numOfCols != other.getNumOfRows()) {
    public int getNumOfRows() { return numOfRows; }
    public int getNumOfCols() { return numOfCols; }
    public abstract int get(int i, int j);
    public abstract void set(int i, int j, int v);
    public abstract Matrix clone();
    public String toString() {
        return Matrix.toString(this);
    }
}
```

```java
public class SparseMatrix extends AbstractMatrix {
   private final Map<Index, Integer> matrix;
    ⋮
   public Matrix clone() {
        return new SparseMatrix(getNumOfRows(),getNumOfCols(), matrix);
    }
}
```

```java
public class DiagonalMatrix extends AbstractMatrix {
   private final int[] diagonal;
    ⋮
   public Matrix clone() {
        return new DiagonalMatrix(getNumOfRows(), diagonal);
    }
}
```

it's time to...

RECAP

a class defines a type & provides an implementation for it

a subclass defines a subtype & provides an implementation for it

a type is just a specification

an abstract class defines a type & provides a partial implementation for it

in java, an interface forces you to define types without an implementation

in scala, a trait allows you to define types without an implementation

in swift, the notion of protocols is similar to traits

a class can inherit from only one class but from multiple traits / interfaces / protocol

in python, this notion has no equivalent due to the absence of static typing